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A New Approach Based on Cumulants for Estimating Financial Regression Models with Errors in the Variables: The Fama and French Model Revisited

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Abstract:
This paper proposes to revisit both the CAPM and the three-factor model of Fama and French (1993) in presence of errors in the variables. To reduce the bias induced by measurement and specification errors, we transpose to the cost of equity an estimator based on cumulants of order three and four initially developed by Dagenais and Dagenais (1997) and later generalised to financial models by Racicot (2003). Our results show that our technique has great and significant consequences on the measure of the cost of equity. We obtain ipso facto a new estimator of the Jensen apha.

Résumé:
Ce papier se propose de réexaminer les modèles du CAPM et de Fama et French (1993) à trois facteurs en présence d'erreurs sur les variables. Pour réduire les biais introduits par les erreurs de mesure et de spécification, nous appliquons au coût du capital un estimateur basé sur les cumulants d'ordre 3 et 4 développé initialement par Dagenais et Dagenais (1997) et généralisé aux modèles financiers par Racicot (2003). Nos résultats montrent que notre technique comporte des conséquences importantes et significatives sur la mesure du coût du capital. Nous obtenons par le fait même un nouvel estimateur de l'alpha de Jensen.

Mots-clefs : Erreurs sur les variables; cumulants; moments supérieurs; variables instrumentales; coût du capital; alpha de Jensen.

Keywords: Errors in the variables; Cumulants; Higher moments; Instrumental variables; Cost of Equity; Jensen alpha. .

JEL classification: C19;C49;G12;G31

1. Introduction[1]

Measurement errors stand as one of the major problem in applied financial econometrics. Errors in the variables may lead to the non convergence of the OLS estimator, very often used in the financial literature, casting doubt on the results. Paradoxically, few theoretical and applied efforts have been made to reduce this important bias[2]. Recently, Dagenais and Dagenais (1997) argued that estimators based on moments of order higher than two "performed better than ordinary least squares estimators in terms of root mean squared errors and also in terms of size of type I errors of standard tests in many typical situations of economic analyses". This calls into question the relevance of financial regressions models ignoring this phenomenon, and especially the cost of equity. As it is acknowledged in the financial literature, both the CAPM and the three-factor model of Fama and French (1992, 1993) may be subject to measurement errors[3]. If these models are the common choices, recent evidence suggests, however, that they do not give a good description of expected returns. This paper looks at this issue and proposes to apply for the first time higher moment estimators to financial time series in a simple and parsimonious framework. More specifically, we analyze the accuracy and the performance of an estimator based on moments of order higher than two in presence of errors in the variables, using both the CAPM and the three-factor model of Fama and French (1993) and generalized later by Carhart (1997).

[1] This paper is based on previous works done by Racicot (2003). For a similar approach, see: Racicot, Théoret et Coën (2006).
[2] Hausman's (1978) instrumental variable test is often ignored in empirical econometrics.
[3] See Campbell et al. (1997).

2. Estimators for linear regression models with errors in the variables

It is well known in the economic literature that errors in the explanatory variables tend to lead to inconsistent ordinary least squares (OLS) estimators in linear financial regression models. As underlined by Dagenais and Dagenais (1997), they lead to more perverse effects related to the confidence intervals of the regression parameters and the increase of the sizes of the type I errors. Many studies (Fuller (1987), Bowden (1984) and Aigner et al. (1984) for example) have suggested the use of instrumental variables to obtain consistent estimators, when information on the variances of these errors is not available. Despite these suggestions, instrumental variables techniques are often neglected (Klepper and Leamer (1984)) and (Pal (1980)).

Following Durbin (1954) and Pal (1970), Dagenais and Dagenais (1997) have introduced new unbiased higher moment estimators showing "considerably smaller standard errors". They have presented an estimator, β_H, which is a linear matrix combination of the generalized version of β_d, Durbin's estimator, and $\beta_{p,}$ Pal's estimator. We intend to underline their main results, and then we propose and apply a higher moment estimator to financial time series.

The multivariate version of β_p and β_d, used to define β_H as instrumental variables are respectively given by:

$$\beta_D = (z_1'x)^{-1} z_1'y \quad \text{where} \quad z_1 = \begin{pmatrix} x_{11}^2 & . & . & x_{1K}^2 \\ x_{21}^2 & x_{22}^2 & . & x_{2K}^2 \\ . & . & . & . \\ . & . & . & . \\ x_{N1}^2 & . & . & x_{NK}^2 \end{pmatrix} \tag{1}$$

x_{ij} are the elements of the matrix x and $x = AX$ where $A = I_N - ii'/N$. The matrix x stands for the matrix X calculated in mean deviation. We use the same for y where $y = AY$.

$\beta_P = (z_2'x)^{-1} z_2'y$ where $z_2' = z_3' - 3D(x'x/N)x'$

$$\text{and } z_3 = \begin{pmatrix} x_{11}^3 & . & . & x_{1K}^3 \\ x_{21}^3 & x_{22}^3 & . & x_{2K}^3 \\ . & . & . & . \\ . & . & . & . \\ x_{N1}^3 & . & . & x_{NK}^3 \end{pmatrix} \tag{2}$$

x and y are in mean deviation, $D(x'x/N)$ is a diagonal matrix $K \times K$.

Thus, β_H is given by:

$$\beta_H = W \begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix} \text{ where } W = (C' S^{-1} C)^{-1} C' S^{-1},\ C = \begin{pmatrix} I_K \\ I_K \end{pmatrix} \tag{3}$$

and S is the covariance matrix for $\begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix}$ under H0.

It is straightforward to demonstrate that β_D and β_P are unbiased estimators.

Now, we apply the GLS method:

$$\begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix} = C\beta + \begin{pmatrix} u_D \\ u_P \end{pmatrix} \text{where } C = \begin{pmatrix} I_K \\ I_K \end{pmatrix},\ u_D = (z_1'x)^{-1} z_1' Au \text{ and } u_P = (z_2'x)^{-1} z_2' Au$$

Thus, we get: $\beta_H = (C' S^{-1} C)^{-1} C' S^{-1} \begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix}$ (4)

β_H is unbiased [4] because β_D and β_P are unbiased under H0.

The application of GLS gives us an estimator which is an optimal linear matrix combination of β_D and β_P. Using the theorem of Theil and Goldberger (1961), it is easy to show that the variance of this estimator will be smaller or equal to the smallest variance of the both estimators β_D and β_P [5].

3. Higher moment estimators applied to simple financial models

3.1 The data[6]

We use the monthly returns of five value-weight industries from January 1927 to December 2002: 912 observations. The five industries chosen are those selected by French on his website: manuf, utils, shops, money and other. Fama and French (1993) have proposed a three-factor model in which a security's expected return depends on the sensitivity of its return to the market return and the returns on two portfolios meant to mimic additional risk factors. The two mimicking portfolios added by Fama and French (1993) are SMB (small minus big), which is the difference between the returns on a portfolio of small stocks and a portfolio of big stocks, and HML (high minus low), the

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[4] For a demonstration see Dagenais and Racicot (1993).

[5] We can easily demonstrate that β_H converges in probability when there are errors in the explanatory variables. Another approach would be to use the artificial regressions method developed by MacKinnon (1992). For a presentation of this approach, see Davidson and MacKinnon (1993). The detailed demonstration is available from the authors upon request.

[6] Data are available from authors upon request or may be downloaded from professor K. French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

difference between the returns on a portfolio of high-book-to-market-equity (BE/ME) stocks and a portfolio of low-BE/ME stocks. Carhart (1997) has extended the model by including a fourth common risk factor, UMD, the momentum factor.

The full-period risk-loadings, market risk premium, SMB, HML and UMD, are from the database available on French's website.

3.2 The models: the CAPM and the four-factor model

First, we use the CAPM developed by Sharpe (1964) and Lintner (1965). The cost of equity, $E(R_i)$, is defined as follows:

$$E(R_i) = R_f + [E(R_M) - R_f]\beta_i \tag{5}$$

where R_f is the risk-free interest rate, $E(R_M)$ is the expected return on the value-weight market portfolio, and β_i, the CAPM risk of stock i:

$$R_i - R_f = \alpha_i + \beta_i[R_M - R_f] + e_i \tag{6}$$

As mentioned by Fama and French (1997) the CAPM does not suffice to explain the expected stock returns. Therefore, Fama and French (1993) have proposed a three-factor model, generalized by Carhart (1997):

$$\begin{aligned} E(R_i) - R_f = \beta_i[E(R_M) - R_f] + \beta_{SMB,i}E(SMB) + \beta_{HML,i}E(HML) \\ + \beta_{UMD,i}E(UMD) \end{aligned} \tag{7}$$

where β_i, $\beta_{SMB,i}$, $\beta_{HML,i}$ and $\beta_{UMD,i}$ are the slopes in the regression

$$R_i - R_f = \alpha_i + \beta_i[R_M - R_f] + \beta_{SMB,i}SMB + \beta_{HML,i}HML + \beta_{UMD,i}UMD + e_i \tag{8}$$

4. Results

The estimations of the β used in the definition of the cost of equity are given in table 1 (for the CAPM) and table 2 (for the four-factor model).

-1- For the ordinary least squares estimator β_L (CAPM):

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i [R_{M,t} - R_{f,t}] + e_{i,t} \quad (9)$$

-2- For the estimator based on sample moments of order higher than two, β_H :

$$R_{i,t} - R_{f,t} = \alpha_{H,i} + \beta_{H,i} [R_{M,t} - R_{f,t}] + \beta_{\hat{w},i} \hat{w}_{1t} + e_{i,t} \quad (10)$$

Our estimator is introduced by $\beta_{H,}$ and $\hat{w}_1$ is a vector obtained using artificial regression technique. β_H stands for a combination of a matrix with instrumental variables, whose variables are highly correlated with the variables included in the vector $[R_{M,t} - R_{f,t}]$ but uncorrelated with *e*. This specification ensures the convergence of our estimator despite the presence of errors in the variables.

Table 1: Higher moments estimators and the CAPM

Manuf	α	β_L	α_H	β_H	$\hat{w}$	**Utils**	α	β_L	α_H	β_H	$\beta\hat{w}$	**Shops**	α	β_L	α_H	β_H	$\beta\hat{w}$
Coefficients	0.045	1.037	0.005	1.050	-0.017	Coef.	0.028	0.819	0.044	0.926	-0.144	Coef.	0.051	1.029	-0.011	1.003	0.036
t-student	1.552	201.01	0.189	103.274	-1.498	t-student	0.233	37.525	0.365	21.594	-2.891	t-student	0.579	65.680	-0.126	32.439	0.999
A. R^2		0.978		0.978		A. R^2		0.607		0.610		A. R^2		0.825		0.825	
Money						**Other**											
Coefficients	0.045	1.140	0.072	1.315	-0.235	Coef.	-0.076	0.875	-0.027	0.811	0.086						
t-student	0.499	70.421	0.817	42.079	-6.475	t-student	-1.223	78.262	-0.428	36.987	3.388						
A. R^2		0.844		0.851		A. R^2		0.870		0.872							

A first glance at table 1 shows that the R^2 for the two models are quite similar. We note a significant increase of the beta for the model corrected for the errors in the variables.

With equation (8) we test the accuracy of the four-factor model using the same method. For the estimator β_H, we use the following regression:

$$R_i - R_f = \alpha_{H,i} + \beta_{H,i}[R_M - R_f] + \beta_{HSMB,i}SMB + \beta_{HHML,i}HML + \beta_{HUMD,i}UMD + \\ \beta_{\hat{w}Mkt,i}\hat{w}_{Mkt,t} + \beta_{\hat{w}SMB,i}\hat{w}_{SMB,t} + \beta_{\hat{w}HML,i}\hat{w}_{HML,t} + \beta_{\hat{w}UMD,i}\hat{w}_{UMD,t} + e_i \quad (11)$$

Table 2 : Higher moments estimators and the four-factor model

	α	β_{LMkt}	β_{LSMB}	β_{LHML}	β_{LUMD}	α_H	β_{HMkt}	β_{HSMB}	β_{HHML}	β_{HUMD}	$\beta\hat{w}_{Mkt}$	$\beta\hat{w}_{SMB}$	$\beta\hat{w}_{HML}$	$\beta\hat{w}_{UMD}$
Manuf														
Coefficients	0.047	1.04806	-0.03158	-0.01845	0.006857	0.017	1.07973	-0.05556	0.04095	0.06496	-0.03678	0.03017	-0.00654	-0.06157
t-student	1.59	184.6514	-3.5758	-2.16003	1.009254	0.563	64.9166	-2.93538	1.37632	2.97283	-2.05610	1.40067	-2.10006	-2.65892
A. R^2		0.978406					0.978578							
Utils														
Coefficients	-0.027	0.800247	-0.13860	0.290589	-0.02539	0.055	0.892689	-0.09379	0.14069	0.028998	-0.09330	-0.06401	0.177317	-0.07579
t-student	-0.226	34.7254	-3.86505	8.376921	-0.92047	0.381	13.13023	-1.21783	1.161917	0.326122	-1.28180	-0.73032	1.39808	-0.80436
A. R^2		0.645349					0.646640							
Shops														
Coefficients	0.188	1.016475	0.163177	-0.28482	-0.06430	0.094	1.138117	-0.10409	-0.24212	0.013294	-0.13607	0.344676	-0.04712	-0.08471
t-student	2.272	63.96164	6.598429	-11.9063	-3.38028	1.155	24.76525	-1.99040	-2.94457	0.220169	-2.75291	5.790663	-0.54713	-1.32395
A. R^2		0.854795					0.859721							
Money														
Coefficients	0.108	1.087737	-0.07159	0.219502	-0.13090	0.0344	1.167122	-0.17610	0.362660	-0.04643	-0.10309	0.130042	-0.17291	-0.07302
t-student	1.264	66.58681	-2.81627	8.926572	-6.69472	0.409	24.42881	-.323905	4.242496	-0.73969	-2.00618	2.101519	-1.93128	-1.09769
A. R^2		0.87223					0.873742							
Other														
Coefficients	-0.084	0.847608	0.074267	0.058208	-0.01845	-0.104	0.660115	0.247716	-0.01252	-0.18943	0.214253	-0.21599	0.080681	0.182322
t-student	-1.321	69.31356	3.902828	3.162165	-1.26057	-1.674	18.76840	6.188943	-0.19900	-4.09941	5.663775	-4.7468	1.224078	3.722912
A. R^2		0.874702					0.880257							

As we can see from the observation of table 1, in three cases to five, the CAPM exhibits significant measurement errors (utils, money and other). The consequences of these results are straightforward for the financial industry using α and β. The use of our new estimator taking into account the errors in the variables, may lead to the reject of an asset while it may be accepted by the standard model with a simple O.L.S. estimator. The second alternative would lead the investor to take the wrong decision and may induce a destruction of value.

If we analyze the effects of measurement errors on the four-factor model, we observe that for four industries there are significant measurement errors. These errors are related to the four risk loadings, casting doubt on this model.

As shown by our results, we may conclude that the three and the four-factor models like the CAPM do not suffice to explain the expected stock returns. In presence of measurement errors, it would be interesting to use our new estimators based on sample moments of order higher than two. Thanks to the convergence, our new estimator is better than the simple and so often used OLS estimator. The bias should asymptotically disappear with our estimator while it will still appear with the OLS estimator.

5. Conclusions

Adapting and applying a new estimator based on sample moments of order higher than two on financial data, we have underlined significant presence of errors in the variables in the CAPM and the four-factor model. Our results have shown that our estimator performs better than the OLS estimator, casting doubt on the accuracy of the CAPM, three and four-factor models of Fama and French (1993) and Carhart (1997). Adding our estimator as suggested above, should improve significantly their accuracy and shed a new light on α and β used in financial economics and by the financial industry.

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