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Abstract:
This paper proposes to revisit both the CAPM and the three-factor model of Fama and French (1993) in presence of errors in the variables. To reduce the bias induced by measurement and specification errors, we transpose to the cost of equity an estimator based on cumulants of order three and four initially developed by Dagenais and Dagenais (1997) and later generalised to financial models by Racicot (2003). Our results show that our technique has great and significant consequences on the measure of the cost of equity. We obtain ipso facto a new estimator of the Jensen alpha.

Résumé:

Mots-clés : Erreurs sur les variables; cumulants; moments supérieurs; variables instrumentales; coût du capital; alpha de Jensen.

Keywords: Errors in the variables; Cumulants; Higher moments; Instrumental variables; Cost of Equity; Jensen alpha.

JEL classification: C19;C49;G12;G31
1. Introduction¹

Measurement errors stand as one of the major problem in applied financial econometrics. Errors in the variables may lead to the non convergence of the OLS estimator, very often used in the financial literature, casting doubt on the results. Paradoxically, few theoretical and applied efforts have been made to reduce this important bias². Recently, Dagenais and Dagenais (1997) argued that estimators based on moments of order higher than two “performed better than ordinary least squares estimators in terms of root mean squared errors and also in terms of size of type I errors of standard tests in many typical situations of economic analyses”. This calls into question the relevance of financial regressions models ignoring this phenomenon, and especially the cost of equity. As it is acknowledged in the financial literature, both the CAPM and the three-factor model of Fama and French (1992, 1993) may be subject to measurement errors³. If these models are the common choices, recent evidence suggests, however, that they do not give a good description of expected returns. This paper looks at this issue and proposes to apply for the first time higher moment estimators to financial time series in a simple and parsimonious framework. More specifically, we analyze the accuracy and the performance of an estimator based on moments of order higher than two in presence of errors in the variables, using both the CAPM and the three-factor model of Fama and French (1993) and generalized later by Carhart (1997).

¹ This paper is based on previous works done by Racicot (2003). For a similar approach, see: Racicot, Théoret et Coën (2006).
² Hausman’s (1978) instrumental variable test is often ignored in empirical econometrics.
³ See Campbell et al. (1997).
2. Estimators for linear regression models with errors in the variables

It is well known in the economic literature that errors in the explanatory variables tend to lead to inconsistent ordinary least squares (OLS) estimators in linear financial regression models. As underlined by Dagenais and Dagenais (1997), they lead to more perverse effects related to the confidence intervals of the regression parameters and the increase of the sizes of the type I errors. Many studies (Fuller (1987), Bowden (1984) and Aigner et al. (1984) for example) have suggested the use of instrumental variables to obtain consistent estimators, when information on the variances of these errors is not available. Despite these suggestions, instrumental variables techniques are often neglected (Klepper and Leamer (1984)) and (Pal (1980)).

Following Durbin (1954) and Pal (1970), Dagenais and Dagenais (1997) have introduced new unbiased higher moment estimators showing “considerably smaller standard errors”. They have presented an estimator, $\beta_H$, which is a linear matrix combination of the generalized version of $\beta_d$, Durbin’s estimator, and $\beta_p$, Pal’s estimator. We intend to underline their main results, and then we propose and apply a higher moment estimator to financial time series.

The multivariate version of $\beta_p$ and $\beta_d$, used to define $\beta_H$ as instrumental variables are respectively given by:
\[ \beta_D = (z_1' x)' z_1' y \text{ where } z_1 = \begin{pmatrix} x_{11}^2 & \ldots & x_{1K}^2 \\ x_{21}^2 & x_{22}^2 & \ldots & x_{2K}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1}^2 & \ldots & \ldots & x_{NK}^2 \end{pmatrix} \] (1)

\[ x_{ij} \text{ are the elements of the matrix } x \text{ and } x = AX \text{ where } A = I_N - ii'/N. \text{ The matrix } x \text{ stands for the matrix } X \text{ calculated in mean deviation. We use the same for } y \text{ where } y = AY. \]

\[ \beta_p = (z_2' x)' z_2' y \text{ where } z_2' = z_3' - 3D(x'x/N)x' \]

\[ \text{and } z_3 = \begin{pmatrix} x_{11}^3 & \ldots & x_{1K}^3 \\ x_{21}^3 & x_{22}^3 & \ldots & x_{2K}^3 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1}^3 & \ldots & \ldots & x_{NK}^3 \end{pmatrix} \] (2)

\[ x \text{ and } y \text{ are in mean deviation, } D(x'x/N) \text{ is a diagonal matrix } K \times K. \]

Thus, \( \beta_H \) is given by:

\[ \beta_H = W \begin{pmatrix} \beta_D \\ \beta_p \end{pmatrix} \text{ where } W = (C' S^{-1} C)^{-1} C' S^{-1}, \quad C= \begin{pmatrix} I_K \\ I_K \end{pmatrix} \] (3)

and \( S \) is the covariance matrix for \( \begin{pmatrix} \beta_D \\ \beta_p \end{pmatrix} \) under H0.

It is straightforward to demonstrate that \( \beta_D \) and \( \beta_p \) are unbiased estimators.

Now, we apply the GLS method:

\[ \begin{pmatrix} \beta_D \\ \beta_p \end{pmatrix} = C \beta + \begin{pmatrix} u_D \\ u_p \end{pmatrix} \text{ where } C = \begin{pmatrix} I_K \\ I_K \end{pmatrix}, \quad u_D = (z_1' x)' z_1' Au \text{ and } u_p = (z_2' x)' z_2' Au \]
Thus, we get: $\beta_H = (C' S^{-1} C)^{-1} C' S^{-1} \begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix}$

(4)

$\beta_H$ is unbiased \footnote{For a demonstration see Dagenais and Racicot (1993).} because $\beta_D$ and $\beta_P$ are unbiased under H0.

The application of GLS gives us an estimator which is an optimal linear matrix combination of $\beta_D$ and $\beta_P$. Using the theorem of Theil and Goldberger (1961), it is easy to show that the variance of this estimator will be smaller or equal to the smallest variance of the both estimators $\beta_D$ and $\beta_P$ \footnote{We can easily demonstrate that $\beta_H$ converges in probability when there are errors in the explanatory variables. Another approach would be to use the artificial regressions method developed by MacKinnon (1992). For a presentation of this approach, see Davidson and MacKinnon (1993). The detailed demonstration is available from the authors upon request.}.

3. Higher moment estimators applied to simple financial models

3.1 The data

We use the monthly returns of five value-weight industries from January 1927 to December 2002: 912 observations. The five industries chosen are those selected by French on his website: manuf, utils, shops, money and other. Fama and French (1993) have proposed a three-factor model in which a security’s expected return depends on the sensitivity of its return to the market return and the returns on two portfolios meant to mimic additional risk factors. The two mimicking portfolios added by Fama and French (1993) are SMB (small minus big), which is the difference between the returns on a portfolio of small stocks and a portfolio of big stocks, and HML (high minus low), the

\footnote{Data are available from authors upon request or may be downloaded from professor K. French’s website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html}
difference between the returns on a portfolio of high-book-to-market-equity (BE/ME) stocks and a portfolio of low-BE/ME stocks. Carhart (1997) has extended the model by including a fourth common risk factor, UMD, the momentum factor.

The full-period risk-loadings, market risk premium, SMB, HML and UMD, are from the database available on French’s website.

3.2 The models: the CAPM and the four-factor model

First, we use the CAPM developed by Sharpe (1964) and Lintner (1965). The cost of equity, $E(R_i)$, is defined as follows:

$$E(R_i) = R_f + [E(R_M) - R_f] \beta_i$$ (5)

where $R_f$ is the risk-free interest rate, $E(R_M)$ is the expected return on the value-weight market portfolio, and $\beta_i$, the CAPM risk of stock $i$:

$$R_i - R_f = \alpha_i + \beta_i [R_M - R_f] + \epsilon_i$$ (6)

As mentioned by Fama and French (1997) the CAPM does not suffice to explain the expected stock returns. Therefore, Fama and French (1993) have proposed a three-factor model, generalized by Carhart (1997):

$$E(R_i) - R_f = \beta_i [E(R_M) - R_f] + \beta_{SMB,i}E(SMB) + \beta_{HML,i}E(HML) + \beta_{UMD,i}E(UMD)$$ (7)

where $\beta_i$, $\beta_{SMB,i}$, $\beta_{HML,i}$ and $\beta_{UMD,i}$ are the slopes in the regression

$$R_i - R_f = \alpha_i + \beta_i [R_M - R_f] + \beta_{SMB,i}SMB + \beta_{HML,i}HML + \beta_{UMD,i}UMD + \epsilon_i$$ (8)
4. Results

The estimations of the $\beta$ used in the definition of the cost of equity are given in table 1 (for the CAPM) and table 2 (for the four-factor model).

-1- For the ordinary least squares estimator $\beta_L$ (CAPM):

$$ R_{i,t} - R_{f,t} = \alpha_i + \beta_i [R_{M,t} - R_{f,t}] + e_{i,t} \quad (9) $$

-2- For the estimator based on sample moments of order higher than two, $\beta_H$ :

$$ R_{i,t} - R_{f,t} = \alpha_H + \beta_H [R_{M,t} - R_{f,t}] + \beta_{\hat{w},i} \hat{w}_t + e_{i,t} \quad (10) $$

Our estimator is introduced by $\beta_H$ and $\hat{w}_1$ is a vector obtained using artificial regression technique. $\beta_H$ stands for a combination of a matrix with instrumental variables, whose variables are highly correlated with the variables included in the vector $[R_{M,t} - R_{f,t}]$ but uncorrelated with $e$. This specification ensures the convergence of our estimator despite the presence of errors in the variables.
### Table 1: Higher moments estimators and the CAPM

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<tr>
<th></th>
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<th>α</th>
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<th>Util</th>
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<th>Money</th>
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<th>αₜₜ</th>
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**Coefficients**

- **α**: Coefficients of the Manuf and Shops categories.
- **βₜₜ**: Coefficients of β for the Manuf and Shops categories.
- **αₜₜ**: Coefficients of α for the Manuf and Shops categories.
- **ŵ**: Coefficients of ŵ for the Manuf and Shops categories.
- **Util**: Utilization coefficients for the Manuf and Shops categories.
- **α**: Coefficients of the Money category.
- **βₜₜ**: Coefficients of β for the Money category.
- **αₜₜ**: Coefficients of α for the Money category.
- **ŵ**: Coefficients of ŵ for the Money category.
- **A. R²**: Adjusted R² for the Manuf and Shops categories.
- **Money**: Other coefficients for the Money category.
- **Other**: Coefficients for the Other category.
A first glance at table 1 shows that the R² for the two models are quite similar. We note a significant increase of the beta for the model corrected for the errors in the variables.

With equation (8) we test the accuracy of the four-factor model using the same method.

For the estimator \( \beta_{H^i} \), we use the following regression:

\[
R_i - R_f = \alpha_{H,i} + \beta_{H,i} [R_M - R_f] + \beta_{HSM}_{i} SMB + \beta_{HML}_{i} HML + \beta_{HUMD}_{i} UMD + \\
\beta_{HMB}_{i} \hat{w}_{MB,i} + \beta_{HSM}_{i} \hat{w}_{SMB,i} + \beta_{HML}_{i} \hat{w}_{HML,i} + \beta_{HUMD}_{i} \hat{w}_{UMD,i} + e_i
\] (11)
Table 2: Higher moments estimators and the four-factor model

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<th>β_SMB</th>
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<td>-0.104</td>
<td>0.660115</td>
<td>0.247716</td>
<td>-0.01252</td>
<td>-0.18943</td>
<td>0.214253</td>
<td>-0.21599</td>
<td>0.080681</td>
<td>0.182322</td>
</tr>
<tr>
<td>A. R²</td>
<td>0.874702</td>
<td>0.880257</td>
<td></td>
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</table>
As we can see from the observation of table 1, in three cases to five, the CAPM exhibits significant measurement errors (utils, money and other). The consequences of these results are straightforward for the financial industry using $\alpha$ and $\beta$. The use of our new estimator taking into account the errors in the variables, may lead to the reject of an asset while it may be accepted by the standard model with a simple O.L.S. estimator. The second alternative would lead the investor to take the wrong decision and may induce a destruction of value.

If we analyze the effects of measurement errors on the four-factor model, we observe that for four industries there are significant measurement errors. These errors are related to the four risk loadings, casting doubt on this model.

As shown by our results, we may conclude that the three and the four-factor models like the CAPM do not suffice to explain the expected stock returns. In presence of measurement errors, it would be interesting to use our new estimators based on sample moments of order higher than two. Thanks to the convergence, our new estimator is better than the simple and so often used OLS estimator. The bias should asymptotically disappear with our estimator while it will still appear with the OLS estimator.

5. Conclusions

Adapting and applying a new estimator based on sample moments of order higher than two on financial data, we have underlined significant presence of errors in the variables in the CAPM and the four-factor model. Our results have shown that our estimator performs better than the OLS estimator, casting doubt on the accuracy of the CAPM, three and
four-factor models of Fama and French (1993) and Carhart (1997). Adding our estimator as suggested above, should improve significantly their accuracy and shed a new light on \( \alpha \) and \( \beta \) used in financial economics and by the financial industry.
References


