ESG UQÀM

Cahier de recherche
2007-06

Chaire d'information financière et organisationnelle

Higher Moments as Risk Instruments to Discard Errors in Variables: The Case of the Fama and French Model

Alain Coën[†]

François-Éric Racicot[††]

Raymond Théoret[†††]

[†] Professeur, Université du Québec (Montréal), (UQAM), 315 Ste-Catherine est, Montréal, Québec, Canada, H3X 2X2. E-mail: coen.alain@uqam.ca. Tél.: (514) 987-3000 poste 5680.
[††] Professeur, Université du Québec (Outaouais), (UQO), 283 Boul. Alexandre Taché, Gatineau, Québec, Canada, J8X 3X7. E-mail: francoiseric.racicot@uqo.ca, Tél: (819) 595-3900 poste 1727.
[†††] auteur de correspondance. Professeur, Université du Québec (Montréal), (UQAM), 315 Ste-Catherine est, Montréal, Québec, Canada. H3X 2X2. E-mail: theoret.raymond@uqam.ca. Tél.: (514) 987-3000 poste 4417.

Les auteurs tiennent à remercier les professeurs Christian Calmès et Jean-Pierre Château pour leurs commentaires utiles. Ils veulent également remercier les participants de la Conférence de la Canadian Economic Association (CEA) tenue à Montréal au printemps 2006 de même que les participants de la Global Finance Conference tenue à Rio de Janeiro au printemps 2006.

Higher Moments as Risk Instruments to Discard Errors in Variables: The Case of the Fama and French Model

Abstract. In this paper, our objective is to test if a new procedure for correcting errors in variables can give a better explanation of returns in the context of Fama and French model. This new procedure uses iterated generalized method of moments with instruments incorporating higher moments of the lagged factors. This method is a synthesis of the techniques of correction for errors in variables based on higher moments estimators and of the financial literature about the new measures of risks which views higher moments as risk factors. The results obtained are superior to correction methods which do not consider higher moments as instruments and they reveal the weaknesses of the selected benchmark, the ordinary least squares method, especially in terms of the alphas and of the signs of the factors.

Key words: Errors in variables; Higher moments; Instrumental variables; Asset pricing
JEL classification: C13; C19; C49; G12; G31.

Les moments supérieurs comme instruments de risque pour éliminer les erreurs de mesure: le cas du modèle de Fama et French

Résumé. Dans cet article, nous voulons vérifier si une nouvelle procédure pour corriger les erreurs de mesure peut fournir une meilleure explication des rendements dans le contexte du modèle de Fama et French. Cette nouvelle procédure recourt à la méthode des moments généralisés pour laquelle les instruments sont les moments supérieurs des facteurs décalés. Cette méthode est une synthèse des techniques de correction des erreurs de mesure basée sur les estimateurs à moments supérieurs et de la littérature financière ayant trait aux nouvelles mesures du risque qui considère les moments supérieurs comme des facteurs de risque. Les résultats obtenus s'avèrent supérieurs aux méthodes de correction qui n'utilisent pas les moments supérieurs comme variables instrumentales et ils révèlent les faiblesses de la méthode des moindres carrés ordinaires, choisie comme point de référence, spécialement du point de vue de l'estimation des alphas et des signes des facteurs.

Mots-clefs: Erreurs sur les variables; Moments supérieurs; Variables instrumentales; Évaluation des actifs financiers.
Classification JEL: C13; C19; C49; G12; G31.

Higher Moments as Risk Instruments to Discard Errors in Variables: The Case of the Fama and French Model

1. Introduction

It is well-known that financial data are contaminated by errors in variables. But this problem is often neglected by researchers in the field of finance. Some papers[1] have proposed correction techniques for errors in variables in a CAPM context but these studies are not numerous and much work must be done.

Following Campbell and MacKinlay (1997), the errors in variables problem in asset pricing models can be addressed in two ways. One (Fama and MacBeth(1973)) is to minimize this problem by grouping stocks into portfolios. The second approach (Shanken (1992)) is to explicitly adjust the standard errors to correct for the biases introduced by the errors in variables. But there is a more recent approach adopted by Kandel and Stambaugh (1995) which uses (GLS) instead of least squares as estimator. But, in this approach, the covariance matrix required to weight the observations must be estimated, which might be a problem.

Durbin (1954), Pal (1980) and more recently Dagenais and Dagenais (1997)[2] have proposed an estimation method to correct errors in variables based on higher moments of the error terms of the equations of variables measured with errors. Racicot (2003) transposed this model to financial models of returns[3]. These higher moments serve as instruments to remove errors in variables. In this article, we will use a variant of this method to correct the errors in variables problem which might plague the well-known augmented Fama and French model.

As will be shown, the recourse to higher moments to eliminate errors in variables opens the doors to a synthesis between the modern asset pricing theory and the financial econometric treatment of the errors in variables. Surely, Durbin(1954), Pal (1980) and Dagenais and Dagenais (1997) did not aim to transpose their technique to asset pricing theory and risk measures. But it is well-admitted now that the first two moments of returns, i.e. the mean and the variance, are largely insufficient to take into account the risk of a portfolio. Huang and Litzenberger (1988), Ingersoll (1987) or either Levy (1992) had mentioned that the paradigm of portfolio selection based on the first two moments of the returns distribution maximizes the expected utility of an agent only in either of the two following situations: his utility function is quadratic or the returns distribution is normal. Of course, these two postulates are violated in the real world. Following this "constat", Samuelson (1970), Rubinstein (1973), Kraus and Litzenberger (1976), Friend and Westerfield (1980) and Sears and Wei (1985) have put the foundations of the approach based on higher moments to price financial instruments. These theoretical developments gave birth to the three-moment and the four-moment CAPM[4].

The theoretical developments related to the use of higher moments as measures of financial risk are linked to other sections of risk theory, but a synthesis is yet to do. The theory of stochastic dominance[5] has a long past. The increasing orders of stochastic dominance add more and more higher moments of the returns distribution to judge if a portfolio is superior to another in terms of stochastic dominance. This theory has given rise to new risk measures as the risk of shortfall which is based on the probability distribution of returns. In the same order of ideas, Scott and Hovarth (1980) have alleged that the odd moments of the returns distribution, like a positive mean and a positive skewness, have positive marginal utility for an investor. The positive even moments, like variance and kurtosis, have for their sake negative marginal utility. These developments constitute the foundations of the modern theory of risk which is in construction.

The aim of this paper is to test if a new procedure of correction of errors in variables based on the iterated GMM can give a better explanation of returns in the context of the Fama and French model. Our contribution will be to use as instruments the higher moments of the predetermined regressors[6] in the equation of Fama and French. These instruments will not only serve as technical tools but also as measures of risks, what will integrate our estimation method in the modern theory of risk analysis. As we will see, our procedure is all the more relevant since the regressors (factors) of the equation of Fama and French are long-short portfolios which are consequently close to those of hedge funds. Indeed, these portfolios which mimic some "market anomalies" are long in a category of returns (e.g. returns of small firms) and short in the opposite category (e.g. returns of big firms). Consequently, these portfolios behave like portfolios of options which incorporate many nonlinearities. Moments higher than two are therefore required to take into account these nonlinearities. Taleb (1997) even suggests using moments of order higher than four to measure the risk of an option[7], odd moments being measures of asymmetry and even moments being measures of convexity[8]. The empirical work will show that our choice of instruments is judicious. Really, the correlation of a regressor with moments higher than two is often much more important than the correlation with classical inferior moments.

This paper is organised as follows. The methodology used to carry our empirical analysis is explained in section 2. First, theoretical background of the problem caused by errors in variables and of the chosen instruments- , which consist in higher moments of the regressors of the model of Fama and French, - is discussed. Second, the method of estimation used to correct the problem of errors in variables in the model of Fama and French is presented. Section 3 describes the data series used in this study and reports the empirical results. A summary and concluding remarks are given in the final section.

2. Methodology

2.1 The biases caused by errors-in-variables

The errors in variables problem is well-known in the econometric literature but is often overlooked in finance. Assume[9] a one independent variable regression model where observed y and x are measured with errors. The true values of these variables are y* and x*. The relations between the observed variables and their true counterparts are:

$$y = y^* + u \qquad (1)$$

$$x = x^* + v \qquad (2)$$

where u and v are error vectors. These error vectors have zero means and their variances are: $E(uu') = \sigma_u^2 I$ and $E(vv') = \sigma_v^2 I$. The vectors u and v are assumed orthogonal.

The exact linear relationship between the two unobserved variables is:

$$y^* = x^* \beta \qquad (3)$$

Because y* and x* are unobservable, we substitute equations (1) and (2) in equation (3):

$$y = x\beta + \varepsilon \qquad (4)$$

where $\varepsilon = v - u\beta$. Consequently, by equation (2), x is correlated with the error term ε, and this creates a bias. To compute it, we resolve equation (4) for $\hat{\beta}$, the estimated value of parameter β, by the method of least squares:

$$\hat{\beta} = (x'x)^{-1}x'y \qquad (5)$$

Substituting equation (4) in equation (5), we obtain:

$$\hat{\beta} = \beta + (x'x)^{-1}x'e \qquad (6)$$

Equation (6) shows that $\hat{\beta}$ is biased. And in large samples, it is not consistent:

$$p\lim \hat{\beta} = \beta\left[1 - \frac{\sigma_u^2}{\sigma_x^2}\right] \qquad (7)$$

where $\sigma_x^2 = \sigma_{x*}^2 + \sigma_u^2$. We can rewrite expression (7) as:

$$p\lim \hat{\beta} = \beta(1-\lambda) \qquad (8)$$

where $\lambda < 1$.

According to equation (8), the problem of errors in variables tend to underestimate $\hat{\beta}$, the degree of underestimation being conditioned by λ. The more λ is near 1, the more the problem of errors in variables is serious. At the limit, λ is 1 and $p\lim \beta = 0$. When only y is plagued with errors of measurement in equation (1), there is no bias because x remains uncorrelated with the innovation of the equation. The problem appears when x is measured with error. That creates a correlation between x and the innovation term and therefore, a bias.

The theory about errors in variables suggests therefore some rules-of-thumb to detect them in a model. The first is that if a coefficient increases significantly in absolute value after correcting the least squares estimation for the problem of errors in variables, there is evidence of an error in the variable to which this coefficient is attached. The second rule-of thumb is that if a coefficient changes sign significantly after correction, there is also presumption of errors in variables. We will use these rules-of-thumbs in the empirical section of this paper.

The usual method to cure a model from its errors in variables disease is to regress the erroneous variables on instruments and to replace the observed value of the variable by its estimated value. The classical instruments are exogenous variables or predetermined variables, i.e. lagged values of exogenous or endogenous variables. But the choice of instrumental variables is a real one. These variables must have a good correlation with the erroneous variable but must be unrelated to the innovation of the estimated model. This problem is especially important in the context of the F&F model, where we must find instruments for differences of returns. Indeed, these returns are quite difficult to explain.

2.2 The choice of instrumental variables to estimate the augmented Fama and French (F&F) model

The augmented F&F (1992, 1993 and 1997) model is a purely empirical model which may be written as:

$$R_{pt} - R_{ft} = \alpha + \beta_1 \left(R_{mt} - R_{ft} \right) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \varepsilon_t \quad (9)$$

with:

$R_{pt} - R_{ft}$: the excess return of a portfolio, R_{ft} being the risk-free return;

$R_{mt} - R_{ft}$: the market risk premium;

SMB: a portfolio which mimics the "small firm anomaly", which is long in the returns of selected small firms and short in the returns of selected big firms;

HML: a portfolio which mimics the "income stock anomaly", which is long in returns of stocks of selected firms having a high (book value/ market value) ratio (income stocks) and short in selected stocks having a low (book value/ market value) ratio (growth stocks);

UMD: a portfolio which mimics the "momentum anomaly", which is long in returns of selected stocks having a persistent upper trend and short in stocks having a persistent downwards trend.

To explain the return of a stock or of a portfolio of stocks, the F&F model adds to the sole factor retained by the CAPM, the market risk premium, three other factors which are supposed to represent market anomalies: the small firm anomaly, the book value to market value anomaly and the momentum anomaly[10].

We postulate that the three mimicking portfolios *SMB*, *HML* and *UMD* are measured with errors. They are thus correlated with the innovation term in equation (9) and the estimators of the parameters of this equation obtained by ordinary least squares (OLS) are consequently biased and not consistent. To purge these coefficients from these biases, we must regress in a first pass the independent variables on instrumental variables. The estimated method used in this paper will be explained below. The problem lays in the choice of these instruments.

As we said before, it is difficult to find valuable instruments for the excess returns of the mimicking portfolios. Being long in some stocks and short in others, their cash flows are similar to those of hedge funds. Higher moments of returns, like asymmetry and kurtosis, might have a great influence on these returns. This suggests the use of higher moments of the variables on the RHS of equation (9) as instrumental variables. An econometric theory is indeed in construction on this subject. Following Durbin (1954) and Pal (1980), Dagenais and Dagenais (1997) showed that higher moments of independent variables of a regression might be valid instruments to remove errors-in-variables. But instead of defining higher moments as in these papers, we will adopt a method more akin to asset pricing theory which defines higher moments of returns by powers of these returns.

The method of asset pricing based on higher moments is not new. Rubinstein (1973) and Kraus and Litzenberger (1976) put the foundations of the three-moment and four-moment CAPM. The three-moment CAPM integrated asymmetry of returns in the analysis while the four-moment CAPM added kurtosis.

Some authors, like Kraus and Litzenberger (1976), uses a general utility function to derive the moment-CAPM. Others uses a Taylor expansion of the utility function which, following Samuelson (1970) and Rubinstein (1973), allows to express utility in terms of the higher moments of returns.

Let us assume that the expected utility of wealth, $E[U(W)]$, is function of the *n* first moments of the distribution of wealth:

$$E[U(W)] = \varphi\left[\overline{W}, \sigma_W, skew_W, kur_W, .., sm_{nW}\right] \qquad (10)$$

with $\overline{W}$ the expected value of wealth; σ_W, the volatility of wealth; $skew_W$, its skewness; kur_W, its kurtosis and sm_{nW}, the n^{th} moment of the distribution of wealth. We incorporate moments of order 5 and more because we know that they might be important to explain the returns of long-short portfolios like mimicking portfolios which are the foundation of the F&F model.

The expected utility of end-of-period wealth is maximized over the one period horizon subject to the constraint of initial wealth, which is:

$$a_0 + \sum_{i=1}^{n} a_i = w_0 \qquad (11)$$

According to equation (11), the initial wealth w_0 is allocated between the risk-free asset a_0 and n other risky assets designated by a_i. To maximize the utility of end-of-period wealth subject to (11), we form the usual Lagrangian:

$$L = E[U(W)] - \phi\left(a_0 - \sum_{i=1}^{n} a_i - w_0\right) \qquad (12)$$

Taking the first-order conditions for a maximum, we have:

$$\frac{\partial L}{\partial a_0} = \varphi_{\overline{W}} \times \frac{\partial \overline{W}}{\partial a_0} - \phi = 0 \qquad (13)$$

$$\frac{\partial L}{\partial a_{i,i\neq 0}} = \varphi_{\overline{W}} \times \frac{\partial E(w)}{\partial a_i} + \varphi_{\sigma W} \times \frac{\partial \sigma_W}{\partial a_i} + \varphi_{skew_W} \times \frac{\partial skew_W}{\partial a_i} + \varphi_{kur_W} \times \frac{\partial kur_W}{\partial a_i} + \dots + \varphi_{sm_{nW}} \frac{\partial sm_{nW}}{\partial a_n} - \phi = 0$$

$$(14)$$

with $\varphi_x = \dfrac{\partial E[U(X)]}{\partial x}$.

End-of-period expected wealth is equal to:

$$\overline{W} = (1 + R_0)a_0 + \sum_{i=1}^{n} [(1 + E(R_i))a_i] \qquad (15)$$

We thus have:

$$\frac{\partial \overline{W}}{\partial a_0} = 1 + R_0 \qquad (16)$$

$$\frac{\partial \overline{W}}{\partial a_i} = 1 + E(R_i) \qquad (17)$$

We can therefore express the moments of wealth in terms of the moments of returns of the portfolio as:

$$\sigma_W = \sum_i a_i \beta_{ip} \sigma_p \qquad (18)$$

$$skew_W = \sum_i a_i \gamma_{ip} skew_p \qquad (19)$$

$$kur_W = \sum_i a_i \theta_{ip} kur_p \qquad (20)$$

$$sm_{nW} = \sum_i a_i \omega_{ip} sm_{np} \qquad (21)$$

with $\sigma_p = \sqrt{E\left(R_p - \overline{R}_p\right)^2}$, the volatility of the return of the risk assets;

$\beta_{ip} = \dfrac{E\left[\left(R_i - \overline{R}_i\right)\left(R_p - \overline{R}_p\right)\right]}{\sigma_p^2}$, the beta of risk asset i with the investor's portfolio of risk assets. $skew_p = \sqrt[3]{E\left[\left(R_P - \overline{R}_p\right)^3\right]}$, the asymmetry of the return of the portfolio and

$\gamma_{ip} = \dfrac{E\left[\left(R_i - \overline{R}_i\right)\left(R_p - \overline{R}_p\right)^2\right]}{sk_p^3}$, the gamma of risk asset i with the investor's portfolio of risk assets . It is easy to generate all the other variables by following this design.

Equating equations (13) and (14) to delete ϕ and taking into account equations (18) to (21), we arrive at an expression for E(r_i) in terms of the moments of the distribution of the return of an investor's portfolio:

$$E(R_i) - R_0 = -\left(\frac{\varphi_{\sigma_W}}{\varphi_{\overline{W}}}\right)\beta_{ip}\sigma_p - \left(\frac{\varphi_{skew_W}}{\varphi_{\overline{W}}}\right)\gamma_{ip} skew_p - \left(\frac{\varphi_{kur_W}}{\varphi_{\overline{W}}}\right)\theta_{ip} kur_p - \ldots - \left(\frac{\varphi_{sm_{nW}}}{\varphi_{\overline{W}}}\right)\omega_{ip} sm_{np} \qquad (22)$$

with $\frac{\varphi_{moment}}{\varphi_{\overline{W}}}$ being an investor's marginal rate of substitution between expected wealth and a specific moment. According to Scott and Hovarth (1980), these marginal rates of substitution are positive for odd moments, like mean and positive asymmetry, and negative for even moments, like variance and kurtosis. From equation (22), odd moments have, ceteris paribus, a negative impact on expected return from the point of view of investors. Even moments have a positive impact because they represent risk.

Moving from equation (22) to the condition of market equilibrium for $E(R_i)$ requires making, according to Kraus and Litzenberger (1976), the strong assumption of homogeneous expectations for investors. Following this assumption, equation (22) becomes, assuming that *p* is the market portfolio:

$$E(R_i) - R_0 = \left[\frac{d\overline{W}}{d\sigma_W}\right]\sigma_m\beta_{im} + \left[\frac{d\overline{W}}{dskew_W}\right]skew_m\gamma_{im} + \left[\frac{d\overline{W}}{dkur_w}\right]kur_m\theta_{im} + \ldots + \left[\frac{d\overline{W}}{dsm_{nW}}\right]sm_{nm}\omega_{in} \quad (23)$$

The terms in brackets in expression (23) are the slopes of the efficient frontiers whose arguments are expected wealth and the respective moment. We obtain finally the n-moment CAPM:

$$E(R_i) - R_f = \psi_1\beta_{im} + \psi_2\gamma_{im} + \psi_3\theta_{im} + \ldots + \psi_n sm_{nm} \quad (24)$$

We might use directly expression (24) to define our instruments for removing errors-in-variables by the methods of higher moments in the F&F model. Suppose we want to correct the mimicking portfolio SMB for errors-in-variables. In the first pass of our regressions, we would regress this variable on the co-moments of the lagged excess return of the market portfolio. The variable SMB corrected for errors-in-variables would be:

$$S\hat{M}B_t = \kappa_0 + \kappa_1\beta_{im,t-1} + \kappa_2\gamma_{im,t-1} + \kappa_3\theta_{im,t-1} + ... + \kappa_n sm_{nm,t-1} \qquad (25)$$

Really, we would have to introduce also the co-moments of the mimicking portfolios. This approach would be laborious and will require computing rolling windows of co-moments[11]. But there is a procedure for simplifying equation (25). Kraus and Lintzenberger (1976)[12] have shown that a three-moment CAPM is consistent with the following quadratic form:

$$R_i - R_0 = \alpha_0 + \alpha_1(R_m - R_0) + \alpha_2(R_m - R_0)^2 \qquad (26)$$

and consequently a n-moment CAPM can be written as:

$$R_i - R_0 = \alpha_0 + \alpha_1(R_m - R_0) + \alpha_2(R_m - R_0)^2 + \alpha_3(R_m - R_0)^3 + ... + \alpha_{n-1}(R_m - R_0)^{n-1} \qquad (27)$$

A test on α_2 is a test on skewness preferences in asset pricing and a test on α_3, a test on kurtosis preferences, and so on. The higher moments are consequently powers of returns in this approach. We therefore use a financial theory, the n-moment CAPM, to give an object to the method of Dagenais and Dagenais (1997) for correcting errors-in-variables. Let us return to the variable SMB, which we want to correct for the problem of errors-in-variables. In the first pass of the regression, this variable will be regressed on:

$$\mathrm{S\hat{M}B_t} = \mathrm{f}\left(\mathrm{F_{it-1}, F_{it-1}^2, F_{it-1}^3, ..., F_{it-1}^5, ...}\right) \qquad (28)$$

where F_i are the variables in the RHS of the equation of Fama and French (equation 9) including SMB. They stand for the higher moments of these variables. F_{it-1}^2 stands for the skewness of factor F_i; F_{it-1}^3, for its kurtosis, and so on.

The variables appearing on the RHS of equation (28) will serve as instrumental variables in an iterated GMM procedure to estimate the F&F model, as explained in the next section. The moments conditions of the GMM estimator will preserve orthogonality between the regressors of the F&F equation and the innovation of this equation.

2.3 The iterated GMM procedure

We want to estimate the F&F model, that is:

$$R_{pt} - R_{ft} = \alpha + \beta_1 (R_{mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \varepsilon_t \qquad (29)$$

by the method of iterated GMM. The benchmark is the OLS procedure because it is the usual method to estimate this model.

The instruments used in this regression are given by equation (28). These are the first lagged value of each regressor in equation (29) which are expressed up to the fifth power. These variables represent the higher moments of the factors of risk in the equation of F&F. We call them: "risk instruments". These instruments are consequently $(R_{mt-1} - R_{ft-1}), (R_{mt-1} - R_{ft-1})^2, \ldots, (R_{mt-1} - R_{ft-1})^5$; $(SMB_{t-1}), (SMB_{t-1})^2, \ldots,$ $(SMB_{t-1})^5$; $(HML_{t-1}), (HML_{t-1})^2, \ldots, (HML_{t-1})^5$; $(UMD_{t-1}), (UMD_{t-1})^2, \ldots, (UMD_{t-1})^5$. To these instrumental variables, we add other macroeconomic and financial variables to explain returns. These variables will be specified in the next section.

A moment condition is defined for each of these instrumental variables. These moments conditions specify that each instrumental variable must be orthogonal to the innovation term in equation (29). Let Z be the matrix of instrumental variables. The orthogonality condition is:

$$E(Z'\varepsilon) = 0 \qquad (30)$$

with $\varepsilon = h(Y, X, \theta)$, θ being the vector of parameters to estimate.

These moments conditions are approximated by their empirical counterpart, the sample moments, that is:

$$\frac{1}{N}\sum_{i=1}^{N} Z_i \varepsilon_i = G(Y, X, Z; \theta) \qquad (31)$$

To estimate the vector of parameters, we must weight the moments by a matrix, which we will define later. By finding the vector θ which minimizes the following function:

$$\arg\min_{\theta} G'(Y, X, Z; \theta)\ A\ G(Y, X, Z; \theta) \qquad (32)$$

we obtain the GMM estimator $\hat{\theta}$ of the vector of parameters θ. In this equation, *A* is the weighting matrix.

According to the GMM method, *A* is the inverse of the covariance matrix of the sample moments. This is equivalent to put less weight on the moments conditions that are more imprecise. That seems quite reasonable. This matrix *A* weights the higher moments of the regressors appearing in equation (29). Instead of computing the higher moments in conformity with the method of Dagenais and Dagenais (1997), we let the matrix *A* weighting optimally these higher moments. That is an advantage of the iterated GMM method which computes the optimal covariance matrix between the moments, the higher moments of the regressors in our case.

There are «grosso modo» two methods to compute the weighting matrix *A*. Let: $\hat{A} = \hat{\Phi}^{-1}$, where $\hat{\Phi}$ is the estimated covariance matrix of the moments conditions. This computed matrix might be the White matrix or the HAC[13] matrix. The White matrix is the following:

$$\hat{\Phi}_{White} = \hat{\Gamma}_0 = \frac{1}{T-k}\sum_{t=1}^{T} G_t^{'} G_t \qquad (33)$$

where T is the number of observations and k, the number of regressors. G is defined by equation (31). As we can see, the White matrix is not iterated.

In this article, we use the HAC matrix to compute the matrix of weights, which is more flexible than the White matrix. The following iteration algorithm computes the optimal covariance matrix of the moments:

$$\hat{\Phi}_{HAC} = \hat{\Gamma}_0 + \left[\sum_{j=1}^{T-1} \kappa(j,q)\left(\hat{\Gamma}_j + \hat{\Gamma}'_j\right) \right] \qquad (34)$$

where κ is the kernel and q, the bandwidth. The White matrix is the seed value of this algorithm.

The kernel weights the covariance so that $\hat{\Phi}$ is positive semi-definite. EViews provides two choices for the kernel: the Bartlett and the quadratic spectral. We use the latter one in our estimations because, contrary to the Bartlett kernel, it is smooth and not truncated. It is equal to:

$$\kappa(j,q) = \frac{25}{12(\pi x)^2} \left[\frac{\sin\left(\frac{6\pi x}{5}\right)}{\frac{6\pi x}{5}} - \cos\left(\frac{6\pi x}{5}\right) \right] \qquad (35)$$

where $x = \frac{j}{q}$.

Without entering into the details, let us say that there are two methods to compute the bandwidth in EViews: the Andrews method and the Variable-Newey-West method. For our estimations, we chose the Andrews method to compute the bandwidth q in equation (35) because it seems to converge more quickly towards the solution. This method assumes that the sample moments follow an autoregressive process of order 1 (AR(1))[14].

3. Empirical results and analysis

The data used for the estimation reported below are drawn from the French's website[15]. They include the monthly returns of five industries from January 1927 to December 2002, that is 912 observations. These industries are: manufactures (manuf)[16], utilities (utils), shops (shops), the monetary sector (money) and the other industries (other). These data include also the factors appearing on the RHS of equation (29). We added one portfolio built on the returns of industries. It is a portfolio consisting of the first principal component of the returns of our five industrial sectors. We call it: the index portfolio.

We assume that all the factors appearing on the RHS of equation (29) contain errors in variables. Consequently, we search instruments for the four regressors of equation (29): the excess market risk premium (mkt_rf) and the three other risk factors: SMB, HML and UMD. Ideally, these instruments must have a good correlation with the factors and must be orthogonal to the innovation of equation (29).

Table 1 presents the descriptive statistics of the monthly excess returns of the five industrial sectors and of the index portfolio. It is well-known that that a large number of equity oriented hedge funds strategies exhibit payoffs resembling to a short position in a put option on the market index[17], and therefore bear significant left-tail risk, risk that is ignored by the commonly used mean-variance framework. We observe in Table 1 that the returns of the sectors which are the object of our analysis bear a very high degree of left-tail risk, their kurtosis being significantly higher than 3, the level of kurtosis associated to the Gaussian distribution.

[Insert Table 1 about here]

Consequently, our industrial sectors are comparable to hedge funds, and that gives more relevance to our approach based on higher moments of the distribution of returns. Their payoffs are comparable with those of short put options which have a high degree of kurtosis.

In Table 1, the sectors are ordered by increasing value of their mean return. We notice that the mean-variance relation is not strictly positive, as it should be on an efficient frontier. The shops sector has the same mean as the manufacturing one but has a higher standard deviation. But if we look at the higher order measures of risks, we notice that manufactures have a much higher kurtosis than shops: they are riskier[18] from this point of view. We have here a case of arbitrage between the multiple dimensions of risk which is not taken into account by the efficient frontier. To complicate the analysis of risk, the shops sector has negative asymmetry, which is bad risk and which tends to increase, everything else constant, its mean return relatively to the manufactures which show positive asymmetry (good risk).

Table 2 gives the correlations of the F&F factors with themselves and their chosen instruments from January 1927 to December 2002. Besides the instruments discussed before, we add other macroeconomic and financial variables which are in conformity with the APT theory: the monthly growth of the industrial production(T1IP); the annual growth of the industrial production (T12IP); the monthly and annual growth of the consumer price index (CPI) and the spread between yields of BBB and AAA bonds (spread)[19].

[Insert Table 2 about here]

According to Table 2, the F&F factors are more or less correlated with classic instruments like the the first lag of the factor or with the macroeconomic variables. On the side of macroeconomic variables, we observe that their correlation with the risk factors located at the head of the columns is quite low. Only the monthly growth of industrial production has a moderate correlation with three of these factors.

On the side of predetermined variables of the risk factors of equation (29), we noticed before that there are many nonlinearities in these mimicking portfolios, which are similar to portfolios of hedge funds. These nonlinearities might be captured by the higher moments of these factors. Corroborating this assumption, we notice at Table 2 that the risk factors are generally more correlated or cross-correlated with the higher moments of the first lag of a risk factor than to the first lag itself. For example, the market risk-premium is more related to the higher moments of HML(-1) than to HML(-1) itself. Indeed, the correlation between mkt_rf and HML(-1) is quite low: 0.05. but it is equal to 0.21 for $HML(-1)^5$, that is the higher moment of HML(-1) of order 5. The same is true for the factor UMD and the higher moments of UMD(-1). Consequently, higher moments of lagged variables may constitute quite good instruments.

In Table 3, we report the estimation of the F&F model (equation 29) on the French sample for the five sectors and for the portfolio index. We used five estimation methods: ordinary least squares (OLS) and four methods incorporating instrumental variables to correct for the errors in variables problem that may be present in the factors. These four instrumental variables (IV) methods are: i) classic two-stage least-squares (TSLS) with instruments including macro variables and simple predetermined values of the regressors; ii) two-stage least squares with higher moments (TSLSHM) adding to the instruments of TSLS the higher moments of the first lags of the regressors up to the fifth order (power); iii) classic iterated GMM (iGMM) including the same instruments as TSLS; iv) iterated GMM (iGMMHM) including the same instruments as TSLSHM. For these regressions, the OLS method is the benchmark, for which the estimated coefficients are biased because of the presence of errors-in-variables. The other methods are techniques for correcting the errors in variables biases. We stated in section 2 our criteria to detect an errors in variables problem. Looking at Table 3, we can state some general conclusions.

[Insert Table 3 about here]

Firstly, it will be a source of concern if the adjusted R^2 decreases significantly after correcting for the errors-in-variables problem. After all, we must use estimated factors instead of the observed ones to remove the errors-in-variables problem. Inappropriate instruments might result in low R^2. But this not the case. The adjusted R^2 of the iGMMHM, which seems to be the best method to attack the problem of errors-in-variables as we will see, are quite similar to those of the OLS regressions performed on the sectors. That is quite reassuring.

Secondly, IV methods other than iGMMHM appear not to be very performing for correcting the errors in variables problem in the model of F&F, including the TSLSHM method which nevertheless uses the same instruments as the iGMMHM one, like the higher moments of the predetermined variables. Excluding the iGMMHM method, there are always less significant variables in the other instrumental methods than in the OLS one. For the manufactures equation, there are three significant factors in the OLS estimation and at most two in the other IV methods of estimation. For the money sector, the performance of IV methods other than iGMMHM is particularly bad. For this sector, there are four significant variables in the OLS estimation and very few in the IV methods other than iGMMHM. For the shops, there are also four significant variables in the OLS estimation. The IV methods other than iGMMHM contain less significant variables than OLS and are quite similar in terms of performance, the factor HML being significant in all these regressions. The returns of the utilities industry appear more difficult to estimate than those of the other groups of industries.

One thus notices that, in general, the IV methods other than the iGMMHM do not perform well to remove errors in variables from the OLS regression. They often make not significant the three factors SMB, HML and UMD which are nevertheless often significant in the OLS regression. Surely, that was not the aim of the exercise.

That is not the case with the iGMMHM method. As we can see in Table 3, the three factors are often more significant in the iGMMHM method than the OLS one. We can apply the criteria stated in section 3 to judge the correction done.

The iGMMHM method generally makes the coefficients of SMB, HML and UMD more significant and more important when *expressed in absolute terms* than in the OLS run. The OLS regressions then tend to underestimate the importance of the three factors whatever the industrial sector, which is one of our criteria to identify the presence of errors in variables. Consequently, there seems to have non negligible errors in variables in the three factors and the iGMMHM method seems to do quite a good job for removing these errors.

We stated other criteria to identify the presence of errors in variables in the preceding section. We said than when the sign of a variable changes significantly when we move from the OLS regression to an IV method, there will be presumption of errors in variables. There are three cases where this appears in Table 3. In the regressions of the manufacturing industry, the coefficient of the factor HML is significantly negative in the OLS regression and significantly positive in the iGMMHM regression, that is after correction for errors in variables. There is therefore evidence of a substantial error in the measurement of the impact of the HML factor. We notice the same situation in the equation of the money sector for the same factor. It is reasonable to think that the HML factor is measured with error because it is the ratio of an accounting variable to a market variable.

In the shops regression, another variable seems to make problem: SMB. Its coefficient is significantly positive in the OLS regression and significantly negative in the iGMMHM regression. There is therefore appearance of a significant errors in variables problem for the SMB variable. We can observe only these three cases of changes of sign in all our regression work when we move from the OLS method to the iGMMHM one.

After these preliminary remarks, we must see the financial consequences of the presence of errors-in-variables in the F&F model. Let us first examine the constant of regressions, generally designated by alpha in asset pricing model. We know that alpha is very important in stock picking strategies. Stocks with significant positive alpha must be bought and stocks with negative alpha must be shorted. We see at Table 3 that the alphas obtained by the iGMMHM method have always the same sign than the OLS one, and this result is quite encouraging because it suggests that the estimation of alpha is robust to the problem of errors-in-variables. But the iGMMHM method gives generally more important and more significant alphas than the OLS method. This result is very important from the point of view of financial practice because the presence of errors in variables would reduce alpha and consequently stock-picking activities. For example, for the money sector, alpha is not significant at the 0.05 level in the OLS estimation. In the iGMMHM estimation, it is not only quite significant but its level is much more important than in the OLS one. The same is true for the shops sector and for the index portfolio which pools the sectors.

Two sectors have negative alpha in the OLS and iGMMHM regressions: the utilities sector and the other one. For the utilities sector, the estimated alpha is more important in absolute value and its degree of significance is also higher in the iGMMHM regression than in the OLS one. That suggests that stocks in this sector must be shorted. The negative alpha of the other sector is not significant and is quite low in absolute value in the two methods of regression.

Because the F&F model is a purely empirical one, there in no theory on the signs of the four factors of this model, except perhaps for the market index whose coefficient is generally positive according to CAPM. When a sector has a positive sign for a factor, we can say that it is exposed to the risk of this factor. When inversely it has a negative sign, we may say that is a hedge against the risk of this factor. This interpretation is correct as far as we consider the factors SMB, HML and UMD as risk factors. If we see them as sources of market inefficiencies, the interpretation of signs is less clear.

Our point here is that when there is a change in the sign of the estimated coefficient of a factor when we move from the OLS to the iGMMHM method, this is very problematic from the point of view of financial practice. It means that without removing errors in variables, an investor will take a bad decision in terms of risk when investing in a sector whose signs of factors are falsified by the errors in variables.

4. Summary and conclusions

The iGMMHM method seems to be performing in removing errors in variables which may plague the estimation of the F&F model. In general, we notice that the OLS method, viewed as benchmark, tends to underestimate, in absolute value, the coefficients of the three factors: SMB, HML and UMD, which is in accord with the econometric theory of errors in variables. In three cases, there was also a significant change of sign of coefficient when we correct for the errors in variables problem. In these three cases, there is thus presumption of an errors in variables disease. The estimated alphas, which are related to stock picking activities, are also generally more significant and more important after correcting for errors in variables. These results are obviously important from the point of view of financial practice.

In this study, we have precisely used higher moments of the distribution of the returns of the mimicking portfolios as instruments to purge the F&F model from its errors in variables. That these instruments are highly related to the risk factors is perfectly normal because the returns of the mimicking portfolios incorporate many nonlinearities. These nonlinearities cannot be captured by a classical CAPM or APT model which postulates a linear relation between the returns to be explained and their risk factors. The Fama and French model is also linear. But the risk factors SMB, HML and UMD take care for the presence of nonlinearities. The good relation between these factors and the chosen instruments in this article, which are the higher moments of these variables, tends to demonstrate these points.

There are by now many articles which put in question the F&F model[20]. Our article is instead an argument in favor of this model. The three factors of the F&F model have their place in the explanation of risk, because they package the many nonlinearities which are present in the distributions of returns. They are not market anomalies as they we considered in the past. They are instead the «reservoirs» of moments and co-moments risks.

References

Artzner, P., F. Delbaen, J. M. Eber and D. Heath, "Coherent measures of risk", Mathematical Finance, 9, 203-228, (1999).

Bertsimas, D., G.J. Lauprete and A. Samarov, "Shortfall as a risk measure : properties, optimization and applications", Journal of Economic Dynamics and Control, 28, 1353-1381, (2004).

Campbell, J.Y., A.W. Lo and C. MacKinlay, C. The Econometrics of Financial Markets. Princeton University Press, 1997.

Carhart, M.M., "On persistence in mutual fund performance", Journal of Finance, 52, 57-82, (1997).

Chung, Y.P., H. Johnson and M.J. Schill, "Asset pricing when returns are nonnormal: Fama French factors vs. higher order systematic co-moments", working paper, (2001).

Coën, A. and F.E. Racicot, "Capital asset pricing models revisited: Evidence from errors in variables", Economics Letters, 95 (3), 443-450, (2007).

Coën, A., F.E. Racicot and R. Théoret, "Hedge Fund Returns, Higher Moments, and Nonlinear Risk" in: Gregoriou, G.N. and D.G. Kaiser (eds). Hedge Funds and Managed Futures: A Handbook for Institutional Investors. Risk Books, 2006.

Dagenais, M.G., "Parameter Estimation in regression models with errors in the variables and autocorrelated disturbances", Journal of Econometrics, 64, 145-163, (1994).

Dagenais, M.G. and D.L. Dagenais, "Higher moment estimators for linear regression models with errors in the variables", Journal of Econometrics, 76, 193-221, (1997).

Durbin, J., "Errors in variables", International Statistical Review, 22, 23-32, (1954).

Fama, E.F. and K.R. French, "The cross-section of expected stock returns", Journal of Finance, 47, 427-465, (1992).

Fama, E.F. and K.R. French, "Common risk factors in the returns on stocks and bonds", Journal of Financial Economics, 33, 3-56, (1993).

Fama, E.F. and K.R. French, "Industry costs of equity", Journal of Financial Economics, 43, 153- 193, (1997).

Fama, E.F. and J.D. MacBeth, "Risk, Return, and Equilibrium : Empirical Tests", Journal of Political Economy, 81, 607-636, (1973).

Ferson, W.E. and D.H. Locke, "Estimating the Cost of Capital Through Time : An Analysis of the Sources of Error", Management Science, 44, 485-500, (1998).

Friend, I. and R. Westerfield, "Co-skewness and capital asset pricing", Journal of Finance, 35, 897-914, (1980).

Harvey, C.R. and A. Siddique, "Conditional skewness in asset pricing tests", Journal of Finance, 55, 1263-1295, (2000).

Huang, C. F. and R.H. Litzenberger. Foundations for Financial Economics. Prentice Hall, Englewood Cliffs, 1988.

Ingersoll Jr., J. Theory of Financial Decision Making. Rowman and Littlefield Publishers, 1987.

Jegadeesh, N. and S. Titman, "Returns to buying winners and selling losers: implications for stock market efficiency", Journal of Finance, 48, 65-91, (1993).

Judge, G.G., W.E. Griffiths, R. Carter Hill, H. Lütkepohl and T.C. Lee. The Theory and Practice of Econometrics. Wiley, 1985.

Kandel S. and R. Stambaugh, "Portfolio Inefficiency and the Cross-Section of Expected Returns", Journal of Finance, 50, p. 157-184, (1995).

Kraus, A. and R. Litzenberger, "Skewness preference and the valuation of risk assets", Journal of Finance, 31, 1085-1100, (1976).

Leibowitz, M. L. and R.D. Henriksson, "Portfolio optimization with shortfall constraints: A confidence-limit approach to maintaining downside risk", Financial Analysts Journal, (March-April 1989).

Levy, H., "Stochastic dominance and expected utility: survey and analysis", Management Science, 38, 555-574, (1992).

Lhabitant, F.S. Hedge Funds : Quantitative Insights. John Wiley & Sons, 2004.

Lim, K.-G, "A New Test of the Three-Moment Capital Asset Pricing Model", Journal of Financial and Quantitative Analysis, 24, 205-216, (1989).

Pal, M., "Consistent moment estimators of regression coefficients in the presence of errors in variables", Journal of Econometrics, 14, 349-364, (1980).

Pastor L. and R. Stambaugh, "Costs of Equity Capital and Model Mispricing", Journal of Finance, 54, 67-121, (1999).

Racicot, F.E. and R. Théoret, "The Beta Puzzle Revisited: A Panel Study of Hedge Fund Returns", Journal of Derivatives and Hedge Funds, 13(2), 125-146 (2007).

Racicot, F.E., R. Théoret and A. Coën, "A New Empirical Version of the Fama and French Model Based on the Hausman Specification Test", Proceedings of the Global Finance Conference, **Best Paper Award**, Melbourne, Australia, (April, 2007).

Racicot, F.E. and R. Théoret, "On Comparing Hedge Fund Strategies Using Higher Moment Estimators for Correcting Specification Errors in Financial Models" in: Gregoriou, G.N. and D.G. Kaiser (eds). Hedge Funds and Managed Futures: A Handbook for Institutional Investors. Risk Books, 2006.

Racicot, F.-E**.** (2003), "On Measurement Errors in Economic and Financial Variables", *in* : Three Essays on the Analysis of Economic and Financial Data, chap.3., Ph.D. thesis (ESG-UQAM), 2003. (published in French).

Rubinstein, M., "The fundamental theorem of parameter-preference security valuation", Journal of Financial and Quantitative Analysis, 8, 61-69, (1973).

Samuelson, P. A., "The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments", Review of Economic Studies, 37, 537-542, (1970).

Scott, R.C. and P.A. Hovarth, "On the direction of preference for moments of higher order than the variance", Journal of Finance, 35, 915-919, (1980).

Sears, R. S. and K.C.J. Wei, "Asset pricing, higher moments, and the market risk premium: A note", Journal of Finance, 40, 1251-1253, (1985).

Shanken, J., "On the Estimation of Beta-Pricing Models", Review of Financial Studies, 5, 1-33, (1992).

Taleb, N. Dynamic Hedging. John Wiley, 1997.

Théoret, R. and F.E. Racicot (2007), "Specification errors in financial models of returns: an application to hedge funds", Journal of Wealth Management, 10 (1), 73-86.

Table 1 *Descriptive statistics of the monthly returns of the five industrial sectors*

	μ	**σ**	**skewness**	**kurtosis**
other_rf	0.46	5.2	0.26	9.71
utils_rf	0.53	5.82	0.15	10.44
indcp_rf	0.62	5.54	0.27	11.83
manuf_rf	0.68	5.81	0.47	11.9
shops_rf	0.68	6.27	-0.05	7.74
money_rf	0.75	6.87	0.67	15.25

A portfolio of these sectors was built on their first principal component (indcp_rf). The sectors are ordered by increasing order of mean return. The period covered is from January 1927 to December 2002.

Table 2 *Correlations between the regressors of the model of Fama and French and chosen instruments*

	MKT_RF	SMB	HML	UMD
MKT_RF	1.00	0.32	0.20	-0.34
SMB	0.32	1.00	0.09	-0.17
HML	0.20	0.09	1.00	-0.41
UMD	-0.34	-0.17	-0.41	1.00
MKT_RF(1)	0.10	0.27	0.04	-0.10
$MKT\_RF(1)^2$	0.13	0.29	0.12	-0.16
$MKT\_RF(1)^3$	0.15	0.30	0.12	-0.17
$MKT\_RF(1)^4$	0.16	0.32	0.14	-0.18
$MKT\_RF(1)^5$	0.15	0.32	0.13	-0.16
SMB(-1)	0.02	0.07	-0.06	0.10
$SMB(-1)^2$	0.06	0.04	0.00	0.00
$SMB(-1)^3$	0.08	0.05	-0.04	0.02
$SMB(-1)^4$	0.07	0.06	-0.03	0.01
$SMB(-1)^5$	0.08	0.07	-0.03	0.02
HML(-1)	0.09	0.13	0.19	-0.15
$HML(-1)^2$	0.18	0.20	0.18	-0.28
$HML(-1)^3$	0.20	0.22	0.20	-0.32
$HML(-1)^4$	0.21	0.20	0.20	-0.35
$HML(-1)^5$	0.21	0.20	0.22	-0.35
UMD(-1)	-0.06	-0.10	-0.05	0.06
$UMD(-1)^2$	0.09	0.11	0.10	-0.19
$UMD(-1)^3$	-0.09	-0.09	-0.06	0.17
$UMD(-1)^4$	0.08	0.07	0.06	-0.17
$UMD(-1)^5$	-0.07	-0.06	-0.04	0.16
T1IP	0.16	0.20	0.11	-0.09
T12IP	0.01	-0.01	0.04	0.04
SPREAD	0.00	0.09	0.03	-0.09
T1CPI	-0.01	-0.02	0.10	-0.01
T12CPI	-0.04	-0.04	0.00	0.03

The instruments include the first lag of the regressors and powers two to five of these lagged regressors. Other instruments are: T1IP: monthly growth of the industrial production; T12IP: annual growth of the industrial production. The same transformations are done on the CPI (consumer price index). The variable spread is the spread between the yields of industrial BBB and AAA bonds.

Table 3 *Comparaison of econometric methods to remove errors in the variables in the Fama and French model.*

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Manuf-rf						
OLS	**0.05**	**1,04**	**-0,03**	**-0,02**	**0,006**	**0.97**
	(1,58)	**(184,65)****	**(-3,57)****	**(-2,76)****	**(1,00)**	**1,71**
TSLS	0,06	0,96	-0,04	0,07	0,002	0,97
	(0,95)	(18,95)**	(-0,72)	(1,18)	(0,04)	1,82
TSLSHM	0,07	0,98	-0,06	0,03	0,001	0,97
	(1,61)	(34,17)**	(-2,64)**	(1,06)	(0,04)	1,78
iGMM	-0,01	1,00	-0,04	0,01	0,06	0,97
	(-0,21)	(17,06)**	(-1,02)	(0,23)	(1,19)	1,78
iGMMHM	**0,04**	**0,95***	**-0,04***	**0,02**	**0,02***	**0,97**
	(1,76)	**(86,25)****	**(-2,37)****	**(4,56)****	**(8,67)****	**1,81**

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Money-rf						
OLS	**0,10**	**1,08**	**-0,07**	**0,21**	**-0,13**	**0,87**
	(1,26)	**(66,58)****	**(-2,81)****	**(8,92)****	**(-6,69)****	**1,89**
TSLS	0,86	1,27	0,11	-0,05	-0,12	0,82
	(0,30)	(8,44)**	(0,74)	(-0,35)	(-0,85)	1,97
TSLSHM	0,06	1,27	-0,10	-0,15	-0,17	0,84
	(0,46)	(15,89)**	(-1,72)	(1,70)	(-2,44)**	1,97
iGMM	0,08	1,21	0,03	0,08	-0,13	0,85
	(0,43)	(7,97)**	(0,24)	(0,55)	(-0,88)	(1,97)
iGMMHM	**0,67**	**1,05***	**-0,26***	**-0,28**	**-0,60***	**0,76**
	(4,77)**	**(13,91)****	**(-4,19)****	**(-2,74)****	**(-7,85)****	**1,91**

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Shops-rf						
OLS	**0,18**	**1,01**	**0,16**	**-0,28**	**-0,06**	**0,85**
	(2,27)**	**(63,96)****	**(6,59)****	**(-11,90)****	**(-3,38)****	**1,82**
TSLS	0,38	1,02	0,15	-0,50	-0,20	0,83
	(2,11)**	(7,79)**	(1,16)	(-3,49)**	(-1,63)	1,82
TSLSHM	0,19	1,07	0,02	-0,33	-0,05	0,84
	(1,56)	(14,69)**	(0,47)	(-4,02)**	(-0,90)	1,81
iGMM	0,36	0,97	0,20	-0,41	-0,18	0,84
	(1,70)	(6,22)**	(1,34)	(-2,56)**	(-1,30)	1,84
iGMMHM	**0,24**	**1,10**	**-0,02**	**-0,43***	**-0,11***	**0,83**
	(3,62)**	**(185,73)****	**(-1,95)****	**(-26,27)****	**(-13,56)****	**1,82**

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Utils-rf						
OLS	**-0,02**	**0,80**	**-0,13**	**0,29**	**-0,02**	**0,64**
	(-0,22)	**(34,72)****	**(-3,86)****	**(8,37)****	**(-0,92)**	**1.80**
TSLS	0,32	1,05	-0,30	-0,15	-0,38	0,48
	(1,06)	(4,80)**	(-1,43)	(-0,65)	(-1,86)	1,94
TSLSHM	0,04	0,86	-0,02	0,14	-0,11	0,62
	(0,23)	(8,05)**	(-0,26)	(1,17)	(-1,21	1,80
iGMM	0,68	0,93	-0,34	-0,23	-0,63	0,39
	(1,96)**	(3,37)**	(-1,37)	(-0,93)	(-2,42)**	1,97
iGMMHM	**-0,19**	**1,09***	**-0,19***	**0,23***	**-0,02**	**0,57**
	(-1,55)	**(9,73)****	**(-5,55)****	**(4,28)****	**(-0,52)**	**1,84**

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Other_rf						
OLS	**-0,08**	**0,84**	**0,07**	**0,05**	**-0,01**	**0,87**
	(-1,32)	**(69,31)****	**(3,90)****	**(3,16)****	**(-1,26)**	**1,91**
TSLS	-0,33	0,84	0,18	0,25	0,17	0,84
	(-2,28)**	(7,85)**	(1,73)	(2,15)**	(1,70)	1,83
TSLSHM	-0,15	0,85	0,13	0,11	0,01	0,87
	(-1,59)	(15,28)**	(3,20)**	(1,81)	(0,31)	1,88
iGMM	-0,30	0,88	0,11	0,24	0,14	0,85
	(-2,12)**	(7,81)**	(1,23)	(2,46)**	(1,39)	1,86
iGMMHM	**-0,02**	**0,89***	**0,11***	**0,05**	**-0,07***	**0,86**
	(-0,31)	**(50,80)****	**(5,35)****	**(1,28)**	**(-3,40)****	**1,90**

coefficient (t)	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	adj-R^2 DW
Indcp_rf						
OLS	**0,04**	**0,96**	**0,0003**	**0,05**	**-0,04**	**0,97**
	(1,41)	**(151,10)****	**(0,039)**	**(5,29)****	**(-6,13)****	**1,75**
TSLS	0,09	1,03	0,02	-0,07	-0,10	0,96
	(1,15)	(17,55)**	(0,44)	(-1,17)	(-1,81)	1,87
TSLSHM	0,04	1,01	-0,004	0,02	-0,06	0,96
	(0,88)	(33,59)**	(-0,20)	(0,66)	(-2,45)**	1,83
iGMM	0,16	0,99	-0,001	-0,03	-0,13	0,96
	(2,03)**	(14,21)**	(-0,01)	(-0,47)	(-2,36)**	1,88
iGMMHM	**0,14**	**0,89***	**-0,01**	**0,13***	**-0,11***	**0,96**
	(4,83)**	**(38,71)****	**(-1,41)**	**(12,04)****	**(-12,09)****	**1,73**

The Fama & French model is regressed monthly from 1927 to 2002 for the five groups of American industries constructed by Fama & French and for an index of these groups (indcp) computed by the method of principal components. The regression methods are : OLS (ordinary least squares) ; TSLS (classic two-stage least-squares with instruments including macro variables and predetermined values of the regressors) ; TSLSHM (two-stage least-squares with higher moments adding to the instruments of TSLS higher moments of the first lag of the regressors up to the fifth power) ; iGMM (classic iterated generalized method of moments including the same instruments as TSLS) ; iGMMHM (iterated generalized method of moments with higher moments including the same instruments as TSLSHM). The first entry of a cell is the estimated coefficient and the second, the t statistic. The cells containing significant coefficients at the 0.05 level have their t statistic followed by two asterisks. An asterisk is placed aside a significant coefficient in the iGMMHM estimation when it has the same sign as in the OLS estimation.

Footnotes

---

[1] The first study on the treatment of errors in variables in the context of asset pricing models was done by Fama and Macbeth (1973). Other following papers on this subject are those of : Shanken (1992), Ferson and Locke (1998) and Pastor and Stambaugh (1999).

[2] See also : Dagenais (1994).

[3] On that matter, see also: Théoret and Racicot (2007) and Coën and Racicot (2007).

[4] On the three-moment and four-moment CAPM, see: Lhabitant (2004), chap. 8; Harvey and Siddique (2000); Lim (1989); Kraus and Litzenberger (1976) and Rubinstein (1973) .

[5] See the survey of Levy (1992) on stochastic dominance.

[6] i.e. lagged regressors.

[7] For example, the fifth moment is the asymmetry sensitivity of the fourth one. The seventh moment is the sign of the convexity changes as the underlying asset moves up or down. For Taleb, moments higher than four are especially important for compound options. See: Taleb (1997), p. 202-204.

[8] For example, kurtosis measures negative convexity, which is bad from the point of view of a risk averter.

[9] This presentation follows quite closely Judge and al. (1985).

[10] The original F&F model contained only the first two anomalies. The momentum anomaly, which is due to Carhart (1997) and Jegadesh and Titman (1993), was introduced subsequently. to form the augmented Fama and French model.

[11] As reported in the next section, we will let the method chosen to estimate the model, the iterated GMM, weight optimally the moments instead of weighting them a priori in an ad hoc fashion.

[12] See also L'Habitant (2004) on this point, chap. 8. Sometimes, the higher order moments are expressed in deviations from the mean. According to this formulation, the four-moment CAPM would be: $E(R_i) - R_0 = \beta_1[E(R_m) - R_0] + \beta_2[R_m - E(R_m)]^2 + \beta_3[R_m - E(R_m)]^3$.

[13] HAC is the acronym of : heteroskedasticity and autocorrelation consistent covariance matrix.

[14] For more details on the technical aspects of the GMM estimation method in EViews, see the EViews User's Guide, version 5.1.

[15] The e-mail of French website is: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[16] In parentheses is given the name of the variable.

[17] A short put option carries the risk of rare but large losses.

[18] Which is bad risk, according to Scott and Hovarth (1980). Let us remind that these authors have proved that the positive odd moments of the distribution of returns of a stock are good risk and the positive even moments are bad risk. Positive even moments are negative convexities, which is a bad dimension for a risk-averter.

[19] These series may be found on the following websites : http://www.federalreserve.gov/releases and ftp.bls.gov/pub.

[20] For example, Chung and al. (2001) observed that SMB and HML become insignificant or less significant when moments or co-moments are taken into account. F&F factors might simply be good proxies for the higher co-moments of the return distribution. But our regression technique which considers co-moments as instruments suggests that they capture these co-moments but also other influences.