ESG UQÀM

Cahier de recherche

2010-09

Chaire d'information financière et organisationnelle

Low-Frequency Components and The Weekend Effect Revisited: Evidence from Spectral Analysis

François-Éric Racicot*
Department of Administrative Sciences
University of Quebec – Outaouais (UQO)
Associate Researcher, Corporate Reporting Chair, ESG-UQAM
Laboratory for Research in Probability and Statistics (LRSP)

Novembre 2010

*Correspondence: Department of Administrative Sciences, University of Quebec – Outaouais (UQO), 101, Saint-Jean-Bosco street, Lucien-Brault Building, Gatineau (Hull), Quebec, Canada, J8X 3X7.
Tel.: (819) 595-3900 ext. 1727
Fax: (819) 773-1803
E-mail: francoiseric.racicot@uqo.ca

Abstract

We revisit the well-known weekend anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Straks, 1986; Connolly, 1989; Giovanis, 2010) using an established macroeconometric technique known as spectral analysis (Granger, 1964; Sargent, 1987). Our findings show that using regression analysis with dichotomous variables, spectral analysis helps establishing the robustness of the estimated parameters based on a sample of the S&P500 for the 1972-1973 period. As further evidence of cycles in financial times series, we relate our application of spectral analysis to the recent literature on low-frequency components in asset returns (Barberis *et al*., 2001; Grüne and Semmler, 2008; Semmler *et al.*, 2009). We suggest investment practitioners to consider using spectral analysis for establishing the 'stylized facts' of the financial time series under scrutiny and for regression models validation purposes.

Keywords: Spectral Analysis; Weekend Anomaly; Financial Cycles; Low-frequency Components; Asset Returns.

JEL classification: C1; G11; G17.

Résumé

Dans cet article, nous revisitons la bien connue anomalie du weekend (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Straks, 1986; Connolly, 1989; Giovanis, 2010) en utilisant une technique macroéconomie bien établie et connue sous le nom d'analyse spectrale (Granger, 1964; Sargent, 1987). Nous trouvons que l'analyse spectrale aide à établir la robustesse des paramètres estimés d'un modèle de régression linéaire basée sur des variables dichotomiques pour un échantillon journalier des rendements du S&P500 sur la période 1972-1973. Afin d'apporter des évidences empiriques supplémentaires de la présence de cycles dans les séries financières, nous relions notre application de l'analyse spectrale à la littérature récente sur la présence de basses fréquences dans ces séries de rendements (Barberis *et al*., 2001; Grüne and Semmler, 2008; Semmler *et al.*, 2009). Nous suggérons aux praticiens de la finance de considérer l'analyse spectrale comme un outil d'aide à la validation de modèles financiers et également comme soutien à l'établissement des 'faits stylisés' des séries financières étudiées.

Mots-clefs : Analyse spectrale; anomalie du weekend; cycles financiers; basses fréquences; rendements d'actifs financiers.

Classification JEL : C1; G11; G17.

1. Introduction

The main purpose of this article is to provide the finance community with an overview of a method known as spectral analysis that is well known among academic economists since the mid 60's (Granger, 1964). Thereafter, it has mainly gained popularity in the field of economic sciences among specialists in macroeconomics (Paquin, 1979; Sargent, 1987). As a matter of fact, it has also been a part of the basic curriculum of specialists in econometrics and macroeconometrics for quite long time (Dhrymes, 1970; Box and Jenkins, 1977; Hamilton, 1994); and more recently, in general applied econometrics (Greene, 2000). Lately, it has also appeared in books related to financial econometrics (Wang, 2003) and general econometrics. However, there is not much research using this method that can be found in the applied finance literature; where it does not seem to have gained the same popularity as in the field of economic sciences.

We thus propose an application well known by financial academics and practitioners that is the weekend anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Straks, 1986; Connolly, 1989[1]; Racicot and Théoret, 2001; Giovanis, 2010). As these authors have shown, the weekend anomaly can be simply tested by using a basic dichotomous regression of the index of the S&P500 for the time period 1970-1973. Moreover, it can be tested for the Monday anomaly by using a Student t test or its associated p-value (the significance of that variable). According to Connolly (1989), this type of financial regression may suffer from several types of misspecifications (autocorrelation, conditional heteroskedasticity[2], etc.). Nevertheless, spectral analysis can be used as further evidence of a cycle in the time series under scrutiny; even if there is an apparently misspecified financial regression model that shows a significant variable related to the problem of the Monday anomaly, as shown in our application. Therefore, we propose using spectral analysis for regression model validation purposes.

In this article, we also discuss the new strand of literature related to the theory of low-frequency components in time series of asset returns. The presumption is that there are important low-frequencies in financial time series of returns (Barberis *et al*., 2001; Grüne and Semmeler, 2008; Semmler *et al*., 2009). We believe that the literature on this theory could

[1]For a discussion of the Weekend Anomaly and January Anomaly and a good list of references on the subject, see: Megginson (1997).

[2]For an introduction on ARCH modelling and other useful nonlinear specifications in finance, see Racicot (2000a, 2003a) .

benefit from a judicious use of spectral analysis due to the fact that it is specifically designed for discovering a priori cycles of unknown length.

2. Methodology

2.1 Regression Models for the Weekend Anomaly

To estimate the Weekend Anomaly, we follow Connolly (1989) and Racicot and Théoret (2001) and use dichotomous variables built on the S&P500 index. The method consists in estimating certain parameters related to the days of the week to evaluate the impact of those days that have the most significant influence on the returns of the index. This effect is also known as the day-of-the-week (DOW) effect (Smirlock and Starks, 1986). As it has been shown by several authors (Gibbons and Hess, 1981; Harris, 1986; Keim, 1983), stock returns tend systematically to fall on Monday, and that is mostly for the time period of 1970-1973. After that period, the effect presumably vanished due to the presence of arbitrageurs (Black, 1993). However, some evidence points towards the fact that there would be also a DOW anomaly in other time period (Giovanis, 2010)[3]. Considering this fact, our aim is to simply illustrate the use of spectral analysis on a well-known 'stylized fact', rather than to debate whether or not there would be a DOW anomaly in other recent financial time series. In the jargon of macroeconomists, the 'stylized facts' are the basic empirical fact (Blanchard and Fisher, 1989) or the Granger (1964, 1966) 'typical shape' of financial time series[4].

The following financial regression is used to estimate the Monday anomaly (e.g. Racicot and Théoret, 2001 or Giovanis, 2010)

$$r_t = \beta_1 m_t + \beta_2 T_t + \beta_3 w_t + \beta_4 th_t + \beta_5 f_t + e_t \qquad (1)$$

[3] Giovanis (2010) uses STAR (Smooth Transition Autoregressive) models to test the DOW effect. Particularly, he estimates the following nonlinear regression: $r_t = \pi_1{}'w_t + \beta_1 D_{MON} + \beta_2 D_{TUE} + \beta_3 D_{WED} + \beta_4 D_{THU} + \beta_5 D_{FRI} + (\pi_2 w_t + \gamma_1 D_{MON} + \gamma_2 D_{TUE} + \gamma_3 D_{WED} + \gamma_4 D_{THU} + \gamma_5 D_{FRI}) F(r_{t-d}; \gamma, c) + u_t$ where the D's are defined as in equation (1) and $w_t = (r_{t-1} \ldots r_{t-j})$ is a vector of explanatory variables. He considers two transition functions: the logistic one, $F(r_{t-d}) = (1 + exp\left[-\gamma\left(\frac{1}{\sigma}\right)(r_{t-d} - c)\right])^{-1}$ and the exponential one: $F(r_{t-d}) = 1 - exp\left[-\gamma\left(\frac{1}{\sigma^2}\right)(r_{t-d} - c)^2\right]$, $\gamma>0$; where r_{t-d} is the transition variable, c is the threshold, and γ is the slope of the transition function. Franses and van Dijk (2000) provide an interesting introduction on this type of regime-switching models of returns. See also Racicot and Théoret (2001).

[4] The Granger 'typical spectral shape' displays a power spectrum with the following characteristics: a smooth peak at very low frequency and an exponential decay afterward (at lower frequencies). Using the words of Sargent (1987), 'the dominant feature of the spectrum of most economic time series is that it generally decreases drastically as frequency increases, with most of the power in the low frequency, high periodicity bands'.

where $r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$ is the return computed using the daily observations of the S&P500 index for the year 72 and 73; m_t, w_t, T_t, th_t , and f_t are dichotomous variables which identifies, respectively, the days of the week : Monday, Tuesday, Wednesday, Thursday and Friday, and e_t is, as usual, an error term. To generate our dichotomous variables, we used the EViews program that appears in Table 1.

Insert Table 1 here

2.2 The Geometric Brownian Motion (GBM) Model

The Geometric Brownian Motion (GBM) is one of the most popular models used in quantitative finance. This model is at the heart of the Black and Scholes (1973) option pricing model. It can be used as a data-generating process and should show the behaviour of the random walk model. In the following discussion, we briefly describe how to obtain a simulated time series of asset returns using this type of financial modelling[5].

Assuming that S_t is the price of a stock S observed at time t, the basic GBM model for the returns of that stock price is given by

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz \qquad (2)$$

where μ and σ are, respectively, the mean and standard deviation of dS/S. The element, dz, is the stochastic part of (2). It is known as the Wiener process and defined as

$$dz = \varepsilon\sqrt{dt} \qquad (3)$$

where $\varepsilon \sim N(0,1)$, is a standard normal distribution and dt, is an infinitesimal time step.

To obtain the empirical counterpart the GBM model, (2) must be discretized in an efficient way. Applying Itô's lemma on (2) and after discretizing the resulting equation, one obtains an efficient model to be simulated. The following equations show how to proceed. For a function $G(x, t)$ that depends on a stochastic variable x, Itô's lemma is given by

$$dG = \frac{\partial G}{\partial x}dx + \frac{\partial G}{\partial t}dt + \frac{1}{2}\frac{\partial^2 G}{(\partial x)^2}\sigma^2 dt \qquad (4)$$

Thus, applying (4) on a function $F = ln(S)$ which does not depend on time t gives

$$dF = \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{(\partial S)^2}\sigma^2 dt \qquad (5)$$

Replacing the derivatives in equation (5) by their analytical results and dS by (2), we obtain

[5]For more information on the subject, see: Racicot and Théoret (2001, 2004, and 2006).

$$dF = dlnS = \frac{dS}{S} = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\, dz \qquad (6)$$

Then, by integrating both sides of (6), we obtain the following exact discretized version of equation (2)

$$r_t = \frac{\Delta S_t}{S_{t-1}} = \left(\mu - \frac{1}{2}\sigma^2\right) \Delta t + \sigma\, \varepsilon\, \sqrt{\Delta t} \qquad (7)$$

To perform our spectral analysis of the data generated by equation (7), we assume a risk neutral universe so that μ can be replaced by the risk-free rate r_f. The power spectrum resulting from the simulation of (7) is presented in Section 3.

2.3 Low-frequency Components in Asset Returns

In this section we briefly discuss some literature on the theory of low-frequency components in asset returns (Barberis *et al.*, 2001; Grüne and Semmler, 2008; Semmler *et al.*, 2009). The presumption of this approach is that there would be long cycles[6] in asset returns, as it is shown by the following model (Semmler *et al.*, 2009)[7]

$$r_t^e = \beta_0 + \beta_1 \sin(w1t) + \beta_2 \cos(w1t) + \beta_3 \sin(w2t) + \beta_4 \cos(w2t) \qquad (8)$$

where $w1$ = 2π/5.2857, $w2$ = 1π/3.3636. By using the Discrete Fourier Transform (DFT) filter to estimate the parameters of equation (8) on an annual sample of equity returns (r_t^e) for the time period of 1929-2000, Semmler *et al.* (2009) have found that those parameters are equal to: 0.0718, -0.0971, 0.0086, 0.0712 and 0.0135, respectively for: β_0, β_1, β_2, β_3 *and* β_4. They applied the same approach on the real interest rate time series for the same time period and frequency using the same model, which is given by

$$r_t = \alpha_0 + \alpha_1 \sin(w1t) + \alpha_2 \cos(w1t) + \alpha_3 \sin(w2t) + \alpha_4 \cos(w2t) \qquad (9)$$

where $w1$ = 2π/24.667, $w2$ = 2π/5.2857. They have found that α_0, α_1, α_2, α_3 and α_4 are equal to, 0.01, 0.0182, -0.014, -0.0133 and -0.0042, respectively.

Our aim here is to make the investment practitioners realize that the studies on low-frequency components in financial time series are also evidence of potentially interesting application of spectral analysis, because this literature relates to portfolio management (Semmler *et al.*, 2009). By analogy with some of the empirical works done by specialists in macroeconomics, spectral analysis techniques could be used as supplementary tools for the works specialized in empirical finance to help to discover potentially underlying cycles in the

[6] A time series is qualified of being cyclical if its covariogram is characterized by (damped) oscillations (Sargent, 1987).

[7] Kaufman (1984), chapter 15, presents a model that shares some similarities with equation (8), that is: $y_t = a_1 cos(\omega_1 t) + b_1 \sin(\omega_1 t) + a_2 \cos(\omega_2 t) + b_2 \sin(\omega_2 t)$.

data. In Section 3, we present the power spectrum of (8) as further evidence of the usefulness of spectral analysis for the investment practitioners.

3. Regression Results and Spectral Density Representation

3.1 Regression Results

By running the EViews code presented in Table 1 and applying ordinary least squares (OLS) to equation (1), we obtain the results displayed in Table 2.

Insert Table 2 here

This table shows that the only significant variable is m_t with an estimated coefficient of -0.0028 (or -0.28 if scaled by 100 as often done in this literature). It is significant at 1% with a p-value of 0.0015 (t-Stat of -3.20). Thus, this regression analysis confirms the fact that Monday is the only day of the week that presents an anomaly. Taking into account this result, we can conclude that Stock returns would decline only on Monday–for that sample of data. It should be noted that the Durbin-Watson statistic gives a result that is not close to the 2.0 value. This might indicate the presence of autocorrelation in the residuals. Furthermore, by running other tests on the residuals, we observe that there seems to be a problem of conditional heteroskedasticity[8]. Nevertheless, spectral analysis helps in confirming our results when applied to this sample. A discussion on this topic is presented in the following section.

3.2 Spectral Analysis

To perform the power spectrum of our data, we use a parametric Yule-Walker algorithm which is based on an estimation of an autoregression of order p, AR(p). More precisely, the power spectrum can be represented by a Fourier transform of the autocovariance function, which is (Hamilton, 1994)

$$s_y(\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma_j e^{-i\omega j} \qquad (10)$$

[8]Giovanis (2010) provides evidence of that fact. Other type of misspecifications might arise when using financial regression models. Racicot (1993, 2000, 2003), Coën and Racicot (2007), Racicot and Théoret (2010a, b) provide a discussion of misspecification tests in similar contexts.

where $\gamma_j = E[(y_t - \mu)(y_{t-j} - \mu)]$ is the autocovariance function, $e^{-i\omega j}$ is the Fourier transform[9], ω represents the frequency, $i = \sqrt{-1}$, a complex number. Applying De Moivre theorem, the Fourier Transform becomes

$$e^{-i\omega j} = cos(\omega j) - i\, sin(\omega j) \qquad (11)$$

Thus, the power spectrum can be simplified to

$$s_y(\omega) = \frac{1}{2\pi}\left[\gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j cos\,(\omega j)\right] \qquad (12)$$

The power spectrum or the power spectral density (PSD) can be estimated using a parametric model which could be either a general ARMA(p, q) model or, as in our case, using an AR(p) model. Thus, the power spectrum for an ARMA(p, q),

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \qquad (13)$$

is given by

$$s_y(\omega) = \frac{\sigma^2}{2\pi}\frac{(1+\theta_1 e^{-i\omega} + \theta_2 e^{-i2\omega} + \cdots + \theta_q e^{-iq\omega})}{(1+\phi_1 e^{-i\omega} + \phi_2 e^{-i2\omega} + \cdots + \phi_p e^{-ip\omega})} \qquad (14)$$

The parameters of equation (13) can be estimated by the method of maximum likelihood (or by the two step least squares approach[10]) and the estimated values substituted in (14). But in our case, this equation is simplified because we are using the basic AR(p) process, which implies that all the θ's are null. A power spectrum is thus a representation of $s_y(\omega)$ as a function of the frequencies $\omega_1, \omega_2, \ldots \omega_M$ with $\omega_j = 2\pi j/n$ for a given time series of length *n.*

Figures 1a and 1b show the power spectrum for the sample of the observations used to estimate equation (1) using, an AR(12) and an AR(52), respectively.

Insert Figure 1a and Figure 1b here

---

[9]The Fourier transform of a time series $\{x_1, x_2, x_3, \ldots, x_n\}$ can be written as: $J(\lambda) = n^{-\frac{1}{2}}\sum_{t=1}^{n} x_t e^{-it\lambda}$, $-\infty < \lambda < \infty$ (Brockwell and Davis, 1991). More precisely (Sargent, 1987), the Riesz-Fischer theorem states that for a sequence of complex numbers $\{c_n\}_{n=-\infty}^{\infty}$, there exists a complex-valued function *f(ω)* defined for real ω's belonging to the interval $[-\pi,\ \pi]$ such that $f(\omega) = \sum_{j=-\infty}^{\infty} c_j e^{-i\omega j}$. The function *f(ω)* is called the Fourier transform of the c_k. An important property of the Fourier transform is that it is an *isometric isomorphism* from $l_2(-\infty,\infty)$ to $L_2[-\pi,\ \pi]$ where l_2, is the space of square summable sequences $\{x_k\}_{k=-\infty}^{\infty}$ and L_2, the space of *square Lebesgue integrable functions*. Both are linear spaces. According to Sargent (1987), the Fourier transform "is a *one-to-one* transformation of points in $l_2(-\infty,\infty)$ into points in $L_2[-\pi,\ \pi]$ that preserves both linear structure (i.e. it is an *isomorphism*) and distance between "points" (i.e., it is an *isometric* mapping)". For more information on this subject, see Sargent (1987), chapter XI.

[10]Gourieroux and Montfort (1990), pp. 228-229, describe a very simple method based on OLS that requires only two steps. Firstly, obtain the estimated residuals from applying OLS of y_t on its lagged values: $\hat{\varepsilon}_t = y_t - \sum_{j=1}^{p} \hat{\phi}_j y_{t-j}$. Secondly, take the lagged values of these estimated residuals then to apply OLS on them and the lagged values of the y_t : $y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \hat{\varepsilon}_{t-1} + \theta_2 \hat{\varepsilon}_{t-2} + \cdots + \theta_{t-q} \hat{\varepsilon}_{t-q}$.

Table 3 displays the MATLAB®[11]commands to be run to obtain Figure 1a and 1b.

Insert Table 3 here

As it can be seen in Figure 1 (Figures 1a and 1b), the smooth peak shown approximately at frequency 0.18/0.25 = 0.72[12] is probably an evidence of the Monday anomaly, since our regression analysis shows a significant coefficient precisely for that explanatory variable. Using the formula $\omega_j = 2\pi j/n$, we obtain the number of days at which a cycle might appear; that is: $\frac{n}{j} = \frac{2\pi}{\omega_j} =$ 8.73 where ω_j is approximately 0.72. This result can be interpreted as follows. At approximately every two stock markets effective weeks of five days (i.e. $2 \times 5 = 10 \approx 9$), there would be one Monday that shows a significant anomaly. From our regression analysis, we have found that there seems to be an anomaly on Monday. Spectral analysis would thus confirm an anomaly but for one Monday over three. By combining spectral analysis with our basic regression model, we are able to provide a more accurate picture of the behaviour of the presumed anomaly.

Figures 2a and 2b also show the power spectrum of the daily S&P500 using a different time period which range from January 2007 to April 2010.

Insert Figure 2 here

What should be seen in Figures 2a and 2b, it is that spectral analysis seems to be able to capture the financial crisis that was recently raging in the U.S. A simple plot of the time series would show that there is a sine-wave with high amplitude starting in 2007 moving to mid 2009, which is not the 'typical Granger shape' found in several economic time series.

In order to help in establishing the stylized facts of that series of observations, we also performed the spectra of the S&P500 index using a different frequency; specifically, the monthly returns for the time period of January 1995 to February 2009. The result appears in Figure 3.

---

[11]MATLAB® is a registered trademark of The MathWorks, Inc. Note that in MATLAB, $\omega = 2\pi f/f_s$ where f_s is the sample size.

[12]We compute this ratio to obtain the relative frequency because we have specified in MATLAB the number of observations using fs = 503. This implies that the frequencies' axis is displayed in KHz. That is why we obtained 0.25 = (503/2)/1000. Note that we have also used an AR(12) and an AR(52) to estimate our power spectrum, taking into account the fact that there is 12 months or 52 weeks in a year. When increasing the order of the AR(p), from p=12 to p=52, we observe a small shift in the spectrum (Figure 1a and Figure 1b), which might slightly change our conclusions.

Insert Figure 3 here

This figure displays much less slope changes in comparison to Figures 1a or 1b and it is very close to the strong white noise (as shown in the following figure). However, we can see a small hump at high frequency (approximately 69 Hz) that seems not to be significant but could have been related, if it was more pronounced, to the well-know January effect. We leave this interesting subject for further research.

The figures presented below are also used to help in establishing the stylized facts of our financial time series. The first of them (Figure 4) shows a stochastic process known as a strong white noise (Gourieroux and Jasiak, 2001), the second of them (Figure 5) shows the Gaussian white noise and the last one (Figure 6) represents the popular lognormal process used in most of the financial applications.

Insert Figure 4 here

In Figure 4, we can see that the power spectrum of a strong white noise is linear with a null slope for all frequencies. We can consider this characteristic as reference in order to help in identifying the data generating process of financial time series; which could be assumed to be a strong white noise process in the presence of such spectrum. Thus, these financial time series might present the property of being unforecastable.

Insert Figure 5 and 6 here

Figures 5 and 6 represent the power spectrums of simulations with, a standard normal random number generator and a lognormal one, respectively. As seen in Figure 5, the spectrum of random number, generated from a normal random number generator, is quite wobbly and even showing some cycles. Since a more linear spectrum could have been expected, we could conclude that some statistical artifact might be detected by the PSD and that this aspect might be the result of an inappropriate random number generator. In fact, it is a simple question of scaling. The waves appearing in figure 5 are not significant at a two standard deviation level[13].

Upon a closer look at the method that is often used to generate normal random deviates, we see a nonlinear structure that might cause the generated variables to behave less smoothly than simple uniform random deviates. For instance, the Box-Muller (1958) transformation (Press *et al.*, 1989 or Benninga, 2008) is a very efficient procedure often used to generate normal deviates based on uniform random variable generator. The following discussion gives a brief description of how to generate normal deviates based on *U(0,1)* deviates. Assume that we want to generate two normal deviates y_1 and y_2 based on x_1 and x_2

$$y_1 = \sqrt{-2lnx_1}cos2\pi x_2 \quad (15)$$

$$y_2 = \sqrt{-2lnx_1}sin2\pi x_2 \quad (16)$$

where x_1 and x_2 are two *U(0,1)* deviates. As equation (15) and (16) show, the Box-Muller transform uses sine and cosine functions. These functions could explain the waves obtained in Figure 5 which are a priori unexpected. Finally, to obtain our normal deviates, based on (15) and (16), write x_1 and x_2 as a function of y_1 and y_2 and then apply a Jacobien transformation. For instance,

$$p(y_1)dy_1 = \left|\frac{dx_1}{dy_1}\right| dy_1 = \frac{1}{\sqrt{2\pi}} e^{-y_1^2/2} dy_1 \quad (17)$$

this equation is the obtained probability density function (p.d.f.) of the well-known normal density based on y_1. Analogously to the Kuznets's transformation (Sargent, 1987), it is possible that the generated data could be some statistical artifact detected by the power spectrum. However, this is not the case here. In fact, it could be argued that this is a desirable property as it generates what is generally observed in financial time series.

In the same way, we might relate Figure 6 to the Kuznets's transformation due to the fact that it shows some similarities. Applying a Kuznets's transformation to a white noise process, one can generate a time series that shows large peaks at low frequencies and small peaks at high frequencies; hence, the time series under scrutiny would seem to be characterized by long swings. These swings might be statistical artifacts that are sometimes induced by the transformation and not a characteristic of the data. Here again, if we compute a two standard deviations band, the apparently large swings are not significant. It is just a question of scaling.

[13] Granger (1964), chapter 4, shows how to build a confidence intervals and tests based on a chi-square distribution. He also gives a table to perform the test. The two standard deviations rule is a rough approximation that might reject peaks that are in fact significant.

In addition, this power spectrum will be use as a base of comparison of any other type of financial time series. We provide the power spectrum as a base of comparison to a simulated Geometric Brownian Motion (GBM), since it is at the hearth of basic option pricing models (e.g. the Black and Scholes (1973) formula's). The spectrum of this simulated stochastic process is shown in Figure 7.

Insert Figure 7 here

In this figure, we can observe that the GBM generates lot of waves at different frequencies showing many cycles, resembling Figures 5 and 6. These cycles could also be tested using a two standard deviation band, the result might be that they are not significant. Note that the popularity of this type of modelling in applied finance is probably due to its ease of implementation. Moreover, it can be easily modified in order to account for other stylized facts like stochastic volatility, jumps, etc. These modifications of the GBM would consequently generate a power spectrum that might show some peaks at some frequencies that could correspond to the financial cycles observed in financial time series.

Figure 8 shows the power spectrum of equation (8). This exercise intends to help the financial practitioners to have an idea of the different shapes of spectrums that one can obtain from financial time series. This type of model generates returns with induced sine-waves, so the resulting spectrum should show some peaks at some frequencies, as we can observe in this figure.

Insert Figure 8 here

The figure shows two pronounced peaks, one approximately at frequency 11 Hz and another one at 14 Hz. We can compute the numbers of years at which the cycle seems to appear using: $\frac{n}{j} = \frac{2\pi}{\omega_j} = 21$ where, ω_j, is approximately 0.3 = 11/36 for the first peak and, $\frac{n}{j} = \frac{2\pi}{\omega_j} = 17$ where, ω_j, is approximately 0.38 = 14/36 for the second one (36 = 72 observations divided by 2). This can be interpreted as follows. For the first peak, we obtain a cycle of approximately 21 years and for the second one, a cycle of approximately 17 years. The first of these cycles can be identified as one of the Kuznet's long wave (20 to 30 years) and, the second as one of the building cycle (15 to 20 years) (Granger, 1966). For this reason, we could conclude that the model proposed by Semmeler *et al*. (2009) generates some well-known stylized facts.

Subsequently (see Figure 9), we observe similar patterns repeating themselves with the higher peaks at a higher frequency and then, the lower peaks at a lower frequency.

Insert Figure 9 here

In addition, Figure 9 shows the two peaks identified in Figure 8 which seem to be positioned at similar range of frequencies.

4. Conclusion

In this article, we try to show the usefulness of some of these techniques for the financial analysts and investment community by proceeding analogously as in Granger (1964, 1966) and Sargent (1987). We intent to establish the 'stylized facts' and the 'typical Granger shape' using popular distributions as base of comparison to the standard financial time series. This exercise was conclusive. Consequently, by using the well-known Monday anomaly (Gibbons and Hess, 1981; Harris, 1986; Smirlock and Starks, 1986; Connolly, 1989; Racicot and Théoret, 2001; Giovanis, 2010), we could confirm that our result from a basic regression with dichotomous variables is in fact significant for the Monday anomaly even if the regression is in itself questionable. Thus, spectral analysis can be seen as supplementary tool for helping to confirm some results; for example, in our case, the presence of a cycle for a particular subsample of returns: the S&P500 index for the period 1970-1973. The January anomaly is another well-known stylized fact in the financial literature (Keim, 1983; Tinic and West, 1989; Maloney and Rogalski, 1989; Fama, 1991, Black, 1993). This anomaly could also be tested by the technique of spectral analysis using an approach similar to the one proposed in this paper. Furthermore, after studying the behaviour of the low-frequency components in equity returns (Semmler *et al.*, 2009), we found spectral analysis quite useful for detecting cycles in data generated by the estimated low-frequency model. Therefore, we conclude that this model is able to generate not only cycles of relevant frequencies, but also two cycles of different length.

Actually, some work has been done by researchers (Racicot and Théoret, 2008) to dynamize Jensen's alpha and beta, using the Kalman filter (Racicot and Théoret, 2007, 2010a) and the conditional models in the context of hedge fund returns. This work intends to improve the basic static models of returns frequently used by investment practitioners and academics to establish the performance of these funds. However, further research might be done on how to use spectral analysis to help identifying the cyclical behaviour of important performance and risk parameters, for instance, in the hedge funds industry.

Finally, another possible avenue of research could be based on the use of the *coherence* measure; which is the analogue of the correlation measure (e.g. Pearson or Spearman correlation coefficient) between pairs of financial time series. Indeed, coherence between hedge funds indices or coherence between the Gross Domestic Product (GDP) and hedge funds indices could be computed to help understanding the co-movements of these important indices. Investment practitioners, like portfolio managers who reallocate their portfolios based on sometime unreliable forecasts, could benefit from a better understanding of these measures in order to help establishing leading or lagging indicators of their financial time series.

Acknowledgements

I am grateful to Michel Blanchette, Jean-Paul Paquin, Juan Salazar, F. Haldun Sarlan and Raymond Théoret for their opinions on spectral analysis; and to Sir Clive W. Granger for his suggestions on the subject.

References

Blanchard, O. and Fisher, S. (1989), *Lectures in Macroeconomics*, MIT Press, Cambridge, MA.

Black, F. (1993), "Beta and return", *Journal of Portfolio Management*, Vol. 20, No. 1, pp. 8-18.

Black, F. and Scholes, M. (1973), "The pricing of options and corporate liabilities", *Journal of Political Economy*, Vol. 81, No.3, pp. 637-654.

Barberis, N., Huang, M. and Santos, R. (2001), "Prospect theory and asset prices", *Quarterly Journal of Economics*, Vol. CXVI, No. 1, pp. 1-53.

Benninga, S. (2008), *Financial Modeling*, MIT Press, Cambridge, MA.

Box, G.E.P. and Jenkins, G.M. (1976), *Time series analysis: Forecasting and control*, Holden-Day, Oakland.

Box, G.E.P. and Muller, M.E. (1958), "A note on the generation of random normal deviates", *The Annals of Mathematical Statistics*, Vol. 29, No. 2, pp. 610-611.

Brockwell, P.J. and Davis, R.. (1991), *Time Series: Theory and Methods*, 2nd ed., Springer-Verlag, New York.

Coën, A. and Racicot, F.E. (2007), "Capital asset pricing models revisited: Evidence from errors in variables", *Economics Letters*, Vol. 95, No. 3, pp.443-450.

Connolly, R.A. (1989), "An examination of the robustness of the Weekend effect", *Journal of Financial and Quantitative Analysis*, Vol.24, No.2, pp. 133-169.

Dhrymes, P.J. (1970), *Econometrics: Statistical foundations and applications*, Harper & Row, New York.

Fama, E. (1991), "Efficient capital markets: II", *Journal of Finance*, Vol. 46, No. 5, pp. 1575-1617.

Franses, P.H. and van Dijk, D. (2000), *Non-Linear Time Series Models in Empirical Finance*, Cambridge University Press, Cambridge.

Gibbons, M.R. and Hess, P. (1981), "Day of the week effects and asset returns", *Journal of Business*, Vol. 54, No. 4, pp. 579-596.

Giovanis, E. (2010), "Smooth transition autoregressive models for the day-of-the-week affect: an application to the S&P 500 index", in Gregoriou, G.N. (Ed.), *The Handbook of Trading*, McGraw-Hill, New York, pp. 233-251.

Gourieroux, C. and Monfort, A. (1990), *Serie Temporelles et Modeles Dynamiques*, Economica, Paris.

Gourieroux, C. and J. Jasiak (2001), *Financial Econometrics*, Princeton University Press, Princeton, NJ.

Granger, C.W.J. (1964), *Spectral Analysis of Economic Time Series*, Princeton University Press, Princeton, NJ.

Granger, C.W.J. (1966), "The typical shape of an economic variable", *Econometrica*, Vol. 34, No.1, pp. 150-161.

Greene, W.H. (2000), *Econometric Analysis*, Prentice Hall, Upper Saddle River, NJ.

Grüne, L. and Semmler, W. (2008), "Asset pricing with loss aversion", *Journal of Economic Dynamic and Control*, Vol. 32, No. 10, pp.3253-3374.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press, Princeton NJ.

Harris, L. (1986), "A transaction data study of weekly and intradaily patterns in stock returns", *Journal of Financial Economics*, Vol. 16, No. 1, pp. 99-117.

Kaufman, P.J.(1984), *Handbook of Futures Markets*, Wiley, New York.

Keim, D.B. (1983), "Size-related anomalies and stock return seasonality: Further empirical evidence", *Journal of Financial Economics*, Vol. 12, No. 1, pp.13-32.

Maloney, K.J. and Rogalski, R.J. (1989), "Call-option pricing and the turn of the year", *Journal of Business*, Vol. 62, No. 12, pp.539-552.

Megginson, W.L. (1997), *Corporate Finance Theory*, Addison-Wesley, New York.
Paquin, J.P. (1979), "Structure industrielle, croissance et cycles de l'emploi régional au Canada", *Ph.D. Thesis*, Department of Economics, University of Ottawa.
Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T. (1989), *Numerical Recipes: The Art of Scientific Computing (FORTRAN Version)*, Cambridge University Press, Cambridge, MA.
Racicot, F.E. (1993), "Techniques alternatives d'estimation et tests en présences d'erreurs de mesure sur les variables explicatives", *M.Sc. Thesis*, Department of Economics, University of Montreal, https://papyrus.bib.umontreal.ca/jspui/bitstream/1866/1076/1/a1.1g638.pdf
Racicot, F.E. (2000a), "Notes on nonlinear dynamics", *Working Paper No. 16-2000*, CRG-ESG.
Racicot, F.E. (2000b), "Estimation et tests en présence d'erreurs de mesure sur les variables explicatives : vérification empirique par la méthode de simulation Monte Carlo", *Working Paper No. 09-2000*, CRG-ESG, http://www.repad.org/ca/qc/uq/uqo/dsa/FERCRGESG2000.pdf
Racicot, F.E. (2003a), "Classical and advanced nonlinear models in empirical finance with application to ultra-high-frequency data", in *Trois essais sur l'analyse des données economiques et fiancières*, chapter 1, Ph.D. Thesis, ESG-UQAM.
Racicot, F.E. (2003b), "Erreurs de mesure sur les variables économiques et financières", in *Trois essais sur l'analyse des données économiques et financières*, chapter 3, *Ph.D. Thesis*, ESG-UQAM.
Racicot, F.E. and Théoret, R. (2001), *Traité d'économétrie financière : modélisation financière*, Presses de l'Université du Québec (PUQ), Québec.
Racicot, F.E. and Théoret, R. (2006), *Finance Computionnelle et gestion des risques : ingénierie financière et application Excel (Visual Basic) et Matlab*, Presses de l'Université du Québec (PUQ), Québec.
Racicot, F.E. and Théoret, R. (2007), "A study of dynamic market strategies of hedge funds using the Kalman Filter", *Journal of Wealth Management*, Vol. 10, No. 3, pp. 94-106.
Racicot, F.E. and Théoret, R. (2008), "Conditional financial models and the alpha puzzle: A panel study of hedge fund returns", *Journal of Wealth Management*, Vol. 11, No.2, pp. 59-77.
Racicot, F.E. and Théoret, R. (2010a), "Hedge fund returns, Kalman filter, and errors-in-variables", *Atlantic Economic Journal*, Vol. 38, No.3, pp. 377-378.
Racicot, F.E. and Théoret, R. (2010b), "Optimal instrumental variables generators based on improved Hausman regression, with an application to hedge fund returns ". *Journal of Wealth Management*, Vol. 13, No. 1, pp. 103-123.
Sargent, T.J. (1987*), Macroeconomic Theory*, 2nd ed., Academic Press, San Diego.
Semmler, W., Grüne, L. and Öhrlein, C. (2009), "Dynamic consumption and portfolio decision with time varying asset returns", *Journal of Wealth Management*, Vol. 12, No.2, pp. 21-47.
Smirlock, M. and Starks, L. (1986), "Day-of-the-week and intraday effects in stock returns", *Journal of Financial Economics*, Vol. 17, No. 1, pp.197-210.
Tinic, S.M. and West, R. (1984), "Risk and return: January versus the rest of the year", *Journal of Financial Economics*, Vol. 13, No. 2, pp.561-574.
Wang, P. (2003), *Financial Econometrics*, Routledge, London.

TABLES

Table 1

An example of EViews code used to generate the dichotomous variables of equation (1)

```
smpl 1 503
genr m72=0
genr t72=0
genr w72=0
genr th72=0
genr f72=0
' Loop until the last observation
 for !i=0 to 502
 if day72(!i+1)=1 then
genr m72(!i)=1
else
genr m72(!i)=0
endif
if day72(!i+1)=2 then
genr t72(!i)=1
else
genr t72(!i)=0
endif
if day72(!i+1)=3 then
genr w72(!i)=1
else
genr w72(!i)=0
endif
if day72(!i+1)=4 then
genr th72(!i)=1
else
genr th72(!i)=0
endif
if day72(!i+1)=5 then
genr f72(!i)=1
else
genr f72(!i)=0
endif
 next
```

Source: Racicot and Théoret (2001)

Table 2

OLS estimation of equation (1)

Variable	Coefficient	s.d.	t-stats	p-value
m_t	-0.0028	0.0009	-3.20	0.0015
tu_t	0.0007	0.0008	0.89	0.3716
w_t	0.0004	0.0008	0.54	0.5871
th_t	0.0006	0.0008	0.73	0.4636
f_t	0.0003	0.0008	0.32	0.7473
R^2	0.02	Akaike crit.		-6.76
Adj-R^2	0.016	Schwarz crit.		-6.72
Sum sqr resi	0.03	Hannan-Quinn crit.		-6.74
Log likeli.	1702.07			
D.W. stat.	1.56			

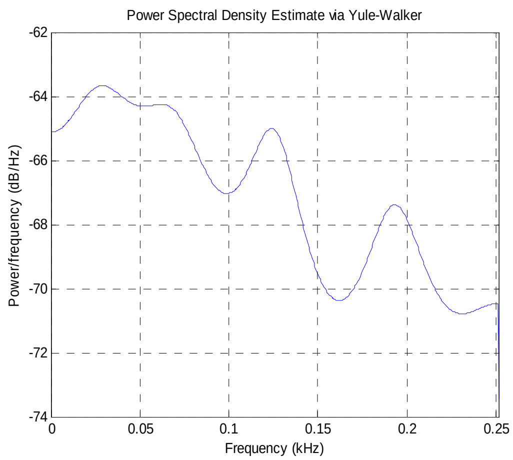
Table 3

MATLAB® commands for generating Figure 1

```
>> load sp500-1970-1973.txt
>> hyulear=spectrum.yulear(12);
>> fs = 503;
>> psd(hyulear,sp500_1970_1973,'Fs',fs)
```

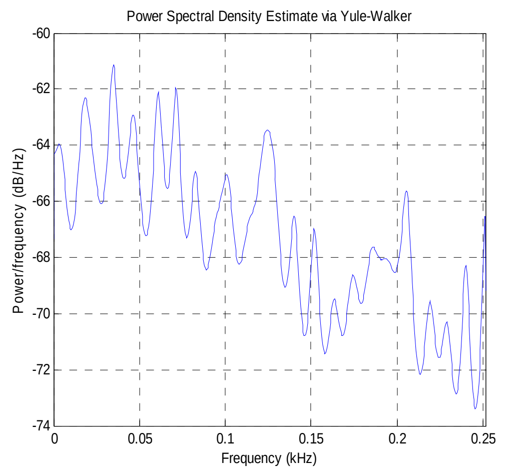
FIGURES

Figure 1a
The Monday Anomaly
Spectrum estimation using an AR(12)
Daily returns of the S&P500 – 1972-1973



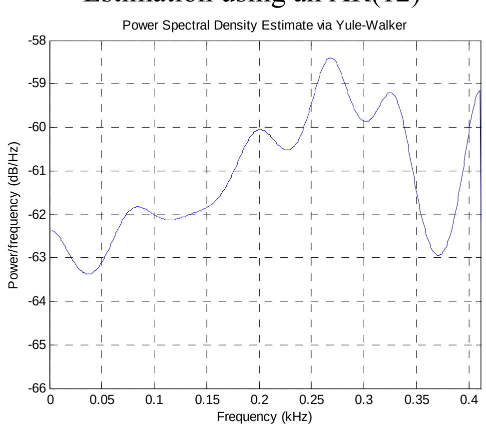
Source : MATLAB®

Figure 1b
The Monday anomaly
Spectrum estimation using an AR(52)
Daily returns of the S&P500 – 1972 - 1973



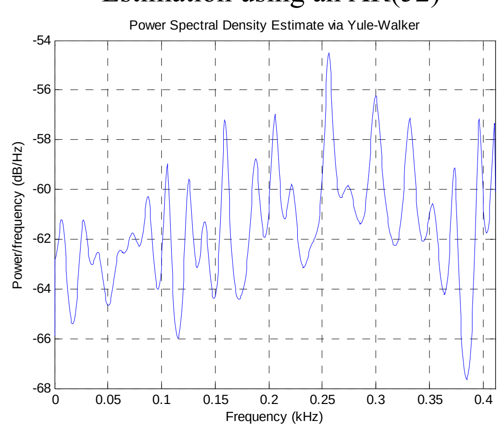
Source : MATLAB®

Figure 2a
Spectrum of the daily S&P500 returns
January 2007 to April 2010
Estimation using an AR(12)



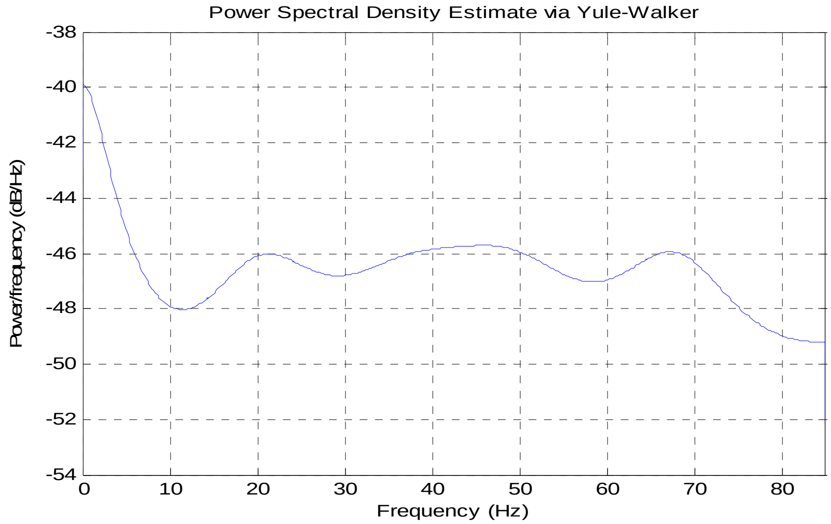
Source : MATLAB®

Figure 2b
Spectrum of the daily S&P500 returns
January 2007 to April 2010
Estimation using an AR(52)



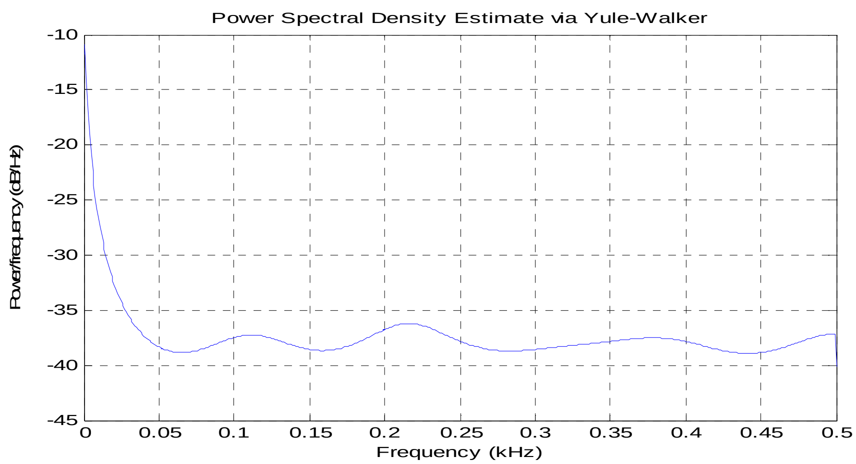
Source : MATLAB®

Figure 3
Power Spectrum of the Montly returns of the S&P500
January 1995 to February 2009
Spectrum estimation using an AR(12)



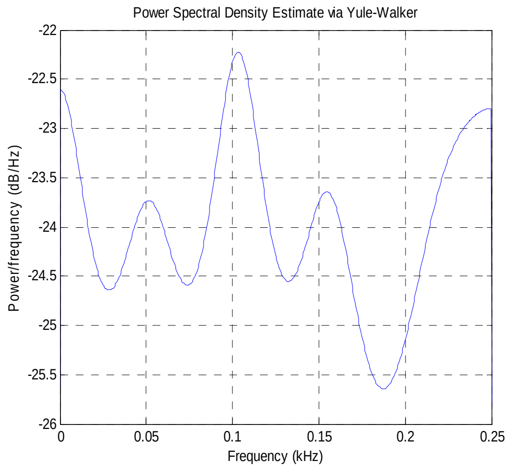
Source : MATLAB®

Figure 4
Spectral density of a strong white noise series



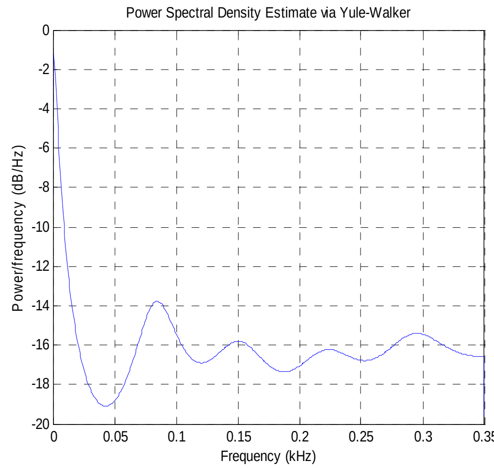
Source : MATLAB®

Figure 5
Spectra of a Normal (0, 1)



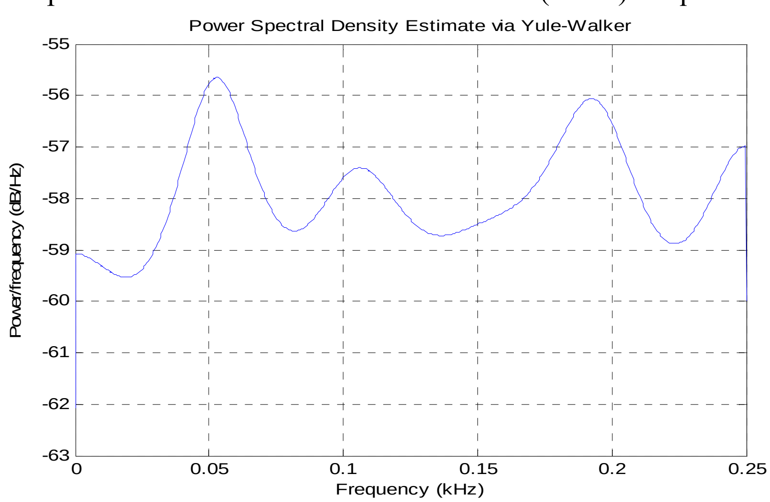
Source : MATLAB®

Figure 6
Spectra of a Lognormal (0, 1)



Source : MATLAB®

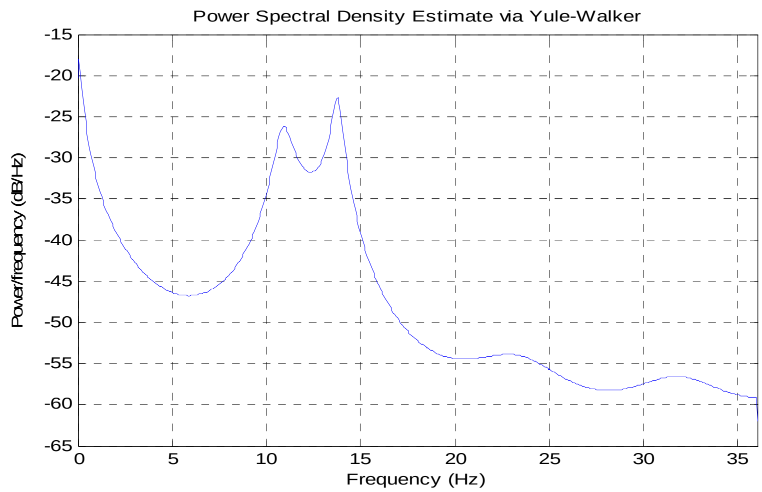
Figure 7
Spectra of a Geometric Brownian Motion (GBM) – equation (7)



Source : MATLAB®

Figure 8

Spectra of a low-frequency model – equation (8)



Source : MATLAB®

Figure 9

Equity returns generated by equation (8)