Abstract

It is well-known that financial institutions follow procyclical risk strategies (Rajan 2005 and 2009, Shin 2009) in the sense that they increase their leverage in economic expansions and reduce it in recessions, which leads to a procyclical behaviour for beta and other risk and financial performance measures. In this paper, we study these risk cycles in the U.S. hedge fund industry resorting to two procedures: conditional modelling and Kalman filtering of Funds alpha and beta. We find that hedge fund betas are usually procyclical. Regarding the alpha, it is usually high at the beginning of a market upside cycle but as the demand pressure stems from investors, it eventually fades away, which suggests that the alpha puzzle is questionable when studied in a dynamic setting.

Key-words: Procyclical risk measures; Aggregate risk; Financial Stability; Conditional models; Kalman Filter; Specification Errors.

JEL classification: C13, C19, C49, G12, G23.
Risk Procyclicality and Dynamic Hedge Fund Strategies

Abstract

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Procyclicité du risque et stratégies dynamiques des fonds de couverture

Résumé

Il est bien connu que les institutions financières suivent des stratégies de risque procycliques (Rajan 2005 et 2009, Shin 2009) en ce sens qu'elles augmentent leur levier lors d'une reprise économique et qu'elles l'abaissent lors d'une récession, ce qui imprime un comportement procyclique au bêta et aux autres mesures de la performance financière. Dans ce papier, nous étudions les cycles du risque dans l'industrie des hedge funds en recourant à deux procédures : la modélisation conditionnelle et le filtre de Kalman. Nos résultats montrent que le bêta des fonds de couverture est généralement procyclique. S'agissant de l'alpaha, celui-ci est généralement élevé au début d'un cycle haussier mais diminue progressivement sous la pression de la demande, ce qui suggère que le puzzle dit de l'alpaha est peut-être un artifact de l'analyse statique utilisée par les chercheurs.

Mots-clés: Mesures de risque procyclique; Risque agrégé; Stabilité financière; Modèles conditionnels; Filtre de Kalman; Erreurs de spécification; Analyse spectrale.

JEL classification: C13, C19, C49, G12, G23.
1. Introduction

Procyclical risk analysis is now one of the main concerns for researchers working in the field of financial institutions, especially in banking research and in macroprudential analysis (Shin 2009, Adrian and Shin 2010). However, these analyses are often cast in a static setting. Moreover, the hedge fund industry, despite its growing share in the world financial system\(^1\), is quite neglected in these analyses of risk procyclicality. For instance, the principal factor having caused the recent financial crisis in the United-States emanated from the subprime mortgage sector where hedge funds are greatly involved. Indeed, according to Adrian and Shin (2010), the share of hedge funds in the origination of US subprime mortgages by the leveraged financial sector was then as high as 32%, which suggests that hedge funds may give raise to important financial shocks having repercussions on the whole economy.

Hedge funds have thus become one of the pillars of the financial system, especially in the United-States. The linkages between the banking sector and the hedge fund industry are so important that a financial crisis emanating from hedge funds may endanger financial stability (e.g. the LTCM episode). Moreover, pension funds savings are increasingly invested in hedge funds, the return on conventional savings vehicles being on a downward trend\(^2\). Hence, the social impact of the bankruptcy of a major hedge fund should not be neglected because it may impair seriously the pensioners’ well-being, as was the case during the last subprime crisis. It is also well-known that a major failure of a financial institution may entail externalities for the whole financial system. Indeed, the

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\(^1\) As an idea of the amount of money invested in the hedge fund world industry, Hull (2006) reports that one trillion $ was invested in this industry in 2004.

\(^2\) According to Devasabai (2010), the share of hedge fund investments in US pension funds, which is presently about 2.5%, will increase to 15% by the end of the decade.
risk taking of a major hedge fund may create aggregate shocks which could give raise to contagion effects. Macroprudential analysis must endogenize these systemic risks\(^3\).

To shed more light on the procyclical risk in the hedge fund industry, we test two hypotheses using data on the Greenwich-Van American hedge fund industry. First, we check the procyclicality of risk management in the hedge fund industry by using two kinds of approaches: the conditional coefficient model and a state-space model based on the Kalman filter\(^4\). We use these setups to study the cyclical behaviour of the hedge funds alphas and betas categorized by strategies. Our methodology allows us to study the cyclical components of hedge fund performance for the whole industry and then by strategy. Financial procyclicality is related to the amplification of aggregate risk by financial institutions over the business cycle frequencies. As hedge fund are known to be very leveraged, there is a suspicion that this procyclical effect exists since the beta is positively related to financial leverage, a ratio which normally increases in economic expansions and decreases in contractions.

The second hypothesis we test concerns the prevalence of the alpha puzzle in a dynamic setting, especially at business cycle frequencies, a problem neglected in the literature (Racicot and Théoret 2008). Indeed, the alpha puzzle is mostly studied in a static setting, which may bias the estimated alpha. We cast the alpha analysis in a dynamic setting to grasp a better understanding of the alpha puzzle. We test if the alpha has also a tendency to be procyclical. In this respect, we expect that the alpha decreases progressively during an economic recovery as the financial assets become increasingly mispriced, the increase in the financial institutions leverage giving way to a wedge between the market price of assets and their fundamentals (Danthine and Donaldson 2002, Geanakoplos 2010). We thus suspect that the resulting fake alphas trend downward with

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\(^3\) For instance, regulators can constrain the financial institutions to pay for the externalities they generate.

\(^4\) In the appendix, we present a method for correcting specification errors in the Kalman filter framework. For more details, see Racicot and Théoret (2010a).
the development of an economic recovery. The puzzle related to the bloated alpha should thus be an artifact of a static analysis. Moreover, this decrease in the alpha during at the end of an economic recovery be used as a leading or forward-looking performance indicator in the hedge fund industry.

The organization of this paper is as follows. In section two, we present the dynamic models of hedge fund returns. Section three provides the empirical results. Section four concludes.

2. Cyclical framework for analyzing hedge fund returns

2.1 Kalman filtering approach

A Kalman filter model is usually made up of an observation or measurement equation and of state or transition equations for the unobserved variables or coefficients\(^5\). To explain the expected hedge fund excess returns, we choose, as measurement equation, the three factor Fama and French (1992, 1993 and 1997) model:

\[
R_{pt} - R_f = \alpha_t + \beta_{ft}(R_{mt} - R_f) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{gprod}gprod_t + \epsilon_t
\]  

(1)

where \(R_{pt} - R_f\) is the excess return of a portfolio, \(R_f\) being the risk-free return; \(R_{mt} - R_f\), the market risk premium; \(SMB\), a portfolio which mimics the "small firm anomaly" which is long in the returns of selected small firms and short in the returns of selected big firms; \(HML\), a portfolio which mimics the "income stock anomaly" which is long in returns of stocks of selected firms having a high (book value/ market value) ratio (value stocks) and short in selected stocks having a low (book value/ market value) ratio (growth stocks);

\(^5\) For an introduction to the Kalman filter and its uses in finance, see: Rachev et al. (2007), chap.11. According to L'Habitant (2004), the Kalman is like a least squares estimation except that the coefficients of the model are updated at every period following the advent of new information. The applications of the Kalman filter in finance might be found in papers related to asset pricing, term structure models and in corporate finance to forecast important ratios like the price-earning ratio [Racicot and Théoret, 2006, chap. 21]. The applications of the Kalman filter in finance go back to the beginning of the eighties. At this time, the filter was used to forecast the real rate of interest and the risk premia in forward and futures markets [Fama and Gibbons, 1982; Hsieh and Kulatilaka, 1982]. Bassett, France & Pliska [1991] resorted to the Kalman filter to forecast forward prices of nontraded securities. There are also extensive applications in the fields of exchange rates and term structure of interest rates where the Kalman filter is used to forecast volatility and other key variables [Pennacchi, 1991]. Finally, there are nevertheless few studies in the hedge fund area to analyse their style dynamics, especially the time varying dimensions of alpha and beta [Swinkels and Van der Sluis, 2001].
The annual industrial production growth rate $g_{prod}$ is the time-varying alpha, $\delta_{t_0}$ the time-varying beta, and $\hat{\epsilon}$ the innovation.

To explain the excess return of a portfolio, Fama and French added to the CAPM market risk premium two other risk factors: the $SMB$ and $HML$ ones. We omit in this equation the momentum factor proposed by Jegadesh and Titman (1993) and Carhart (1997) because the influence of this factor is weak for the majority of hedge fund strategies, a Kalman Filter representation of a process having to be parsimonious.$^6$

According to Capocci and Hübner (2004), a positive sign for $SMB$ in equation (1) suggests that a portfolio manager, here a hedge fund, prefers the stocks of small firms over the stocks of larger ones, which is usually the case for hedge funds. Moreover, a positive sign for the variable $HML$ would be symptomatic of a preference for stocks with a high book-to-market value ratio over stocks with a low book-to-market value ratio, which is also a frequent preference in the hedge fund industry.$^7$

The transition equations give the representation of the time-varying alpha ($\hat{\alpha}$) and beta ($\hat{\delta}_{t_0}$)$^8$. We first assume that the state variables follow a pure random walk and then we extend this recursive process with financial variables which convey valuable information to the hedge fund manager.

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$^6$ While the Fama and French model is still a popular choice to model hedge fund returns, other more parsimonious models have appeared recently in the literature, for instance the one proposed by Berkelaar et al. (2009). These authors suggest that hedge fund strategies could be modeled using the following parsimonious equation: $r_{h,t} = \alpha_{h,t} + \beta_{h,t}^{EQ} r_{EQ,t} + \beta_{h,t}^{FI} r_{FI,t} + \epsilon_t$. This simple two-factor model uses the returns on equity ($r^{EQ}$) and those on fixed income ($r^{FI}$) securities, as many studies have shown that a significant portion of hedge fund return strategies are well explained by these two factors. The parameter $\alpha_{h,t}$ represents the time-varying alpha of strategy $h$ and the other components represent the systematic beta components. Thus, the performance at every time period of a particular strategy can be explained by the manager skills (alpha), the returns on equity (stocks) and the returns on fixed income securities (e.g. bonds). Because of the time-varying nature of their model, they rely on the Kalman Filter to estimate the model parameters.

$^7$ The signs of the factor loadings are seldom discussed in the studies of the F&F model. This gap must be filled because the signification of these signs is partly a matter of interpretation.

$^8$ For another approach to dynamize alphas and betas, see Mamaysky et al. (2007, 2008).
2.1.1 A pure recursive model

We start our modelization with a purely recursive representation of the conditional coefficients of equation (1) which is the simplest way for making coefficients time-varying. This is the model usually used in the Kalman filter literature. Their respective equation is therefore:

\[ \alpha_t = \alpha_{t-1} + \xi_t \]  
\[ \beta_t = \beta_{t-1} + \nu_t \]

with \( \xi_t \) and \( \nu_t \) being respectively the innovations terms of equations (2) and (3). If \( \xi_t \sim iid(0,1) \) and \( \nu_t \sim iid(0,1) \), which amount to pure white noise variables, these equations would represent pure random walk processes.

The filtering of the conditional coefficients in the framework of a pure recursive process is easy to understand. For instance, in equation (3), the coefficient estimated at period \( t-1 \) serves as a seed value (a guess) for the estimation of the coefficient at time \( t \). But the filtered coefficient at time \( t \) is computed optimally with the Kalman filter following the flow of new information which piles up from one period to the next (Ljungqvist and Sargent 2004). Therefore the estimated coefficient \( \hat{\beta}_{t,t} \) may be quite different from the coefficient estimated one period earlier, that is \( \hat{\beta}_{t,t-1} \), even in a pure random walk setting.

2.1.2 Recursive model combined with conditioning financial market variables

Economic and financial information may be added in equations (2) and (3) by relying on a more elaborate recursive process. These variables will allow following the

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9 In these cases, the innovations are white noise.
reaction of the conditional coefficients to the conditioning market information. In this model, the filtered conditional alpha and beta become:

\[ \alpha_t = \alpha_{t-1} + \varphi_1 r_{t-1} + \varphi_2 mkt_{t-1} + \xi_t \]  

(4) \[ \beta_t = \beta_{1,t-1} + \phi_1 r_{t-1} + \phi_2 mkt_{t-1} + \phi_3 VIX_{t-1} + \nu_t \]  

(5)

with \( r \), the level of short-term interest rate; \( mkt \), the market risk premium, that is the spread between the return of the market portfolio and the risk-free return, and \( VIX \), the implicit volatility of the S&P500 index. The conditioning variables are lagged on period, our aim being to track the reaction of the conditional coefficients to the conditioning market information. The retained financial variables, which are lagged one period, are thus known at time \( t \).

Notice that equation (4), like equation (5), might be written in first differences. For instance, equation (4) might be written as:

\[ (\alpha_t - \alpha_{t-1}) = \varphi_1 r_{t-1} + \varphi_2 mkt_{t-1} + \xi_t. \]  

(6)

That means that the revision of the conditional alpha from one period to the other is function of three elements: the interest rate observed one period earlier, the market risk premium also observed with a one period lag and an innovation. The coefficients \( \tilde{\alpha}_1, \tilde{\alpha}_2 \) and the innovation result from the searching process of the filter.

Let us specify the expected sign of the variables incorporated in equations (4) and (5). An increase in the interest rate might be perceived as good news or bad news by hedge funds. If this increase is seen as a forthcoming deterioration of the stock market trend or as an indicator of inflation, that is bad news. But for hedge funds which follow call-like option strategies, that might be good news. The signs of \( \tilde{\alpha}_1 \) and \( \tilde{\alpha}_2 \) are thus indeterminate in equations (4) and (5). As we will see in the empirical section of this paper, this sign is related to the specific hedge fund strategies.
In equation (4), an increase in the market risk premium at time $t-1$, which could be an indication of a market strengthening, may induce hedge funds to position themselves for an increase in their alpha, this positioning being dependent on the portfolio manager skills. In this case, the sign of $\varphi_2$ is positive. But if the alpha is not manageable, this coefficient should not be different from 0.

But this should not be the case for the conditional beta which is viewed as a control or decision variable. If the market risk premium follows a martingale process, we can write:

$$E(mkt_t / \Omega_t) = mkt_{t-1} \quad (7)$$

with $\Omega_t$ the information set.

According to equation (7), an increase of the market risk premium at time $t-1$ is viewed as a strengthening of the stock market. That should encourage hedge funds to take more risk and therefore to increase their beta. The sign of $\varphi_4$ would thus be positive in equation (5). However, Ferson and Schadt (1996) observe what seems to be a beta puzzle in the mutual fund industry, that is a negative link between their beta and the market risk premium. This behaviour seemed perverse to them. Our study will allow to check if there is such a beta puzzle in the hedge fund industry.

In addition to the standard variables usually incorporated in models with time-varying coefficients, we also include the implicit volatility of the S&P500, i.e. the VIX, to explain the conditional beta. Indeed, an increase in market volatility should usually induce hedge funds to bear less risk, and therefore to decrease their beta. However, an increase in market volatility might be welcomed by some hedge fund strategies which are very involved in hedging activities. Another specification not used in this paper to introduce volatility in the beta equation of conditional beta but often found in the literature on market timing consists in using the square of the market risk premium as an
indicator of stock market volatility, which is the second moment of the market risk
premium\(^{10}\):

\[
\beta_t = \beta_{t-1} + \varphi_1 r_{t-1} + \varphi_2 mkt_{t-1} + \varphi_3 mkt_{t-1}^2 + \nu_t \tag{8}
\]

Nevertheless, note that, according to Treynor and Mazuy (1966), the squared market risk
premium might serve to detect good or bad market timing\(^{11}\), a good market timing being
associated to a positive sign for this variable. The sign of the squared market risk premium
is thus theoretically indeterminate in equation (8).

2.2 *Recursive model combined with conditioning financial market variables using the
conditional approach*

There is another way to compute time-varying coefficients. Following the
conditional approach (Christopherson et al. 1998), time-varying alpha and beta are
obtained by substituting equations (4) and (5) in equation (1), that is:

\[
R_{p_t} - R_{p_t} = \alpha_{t-1} + \varphi_1 r_{t-1} + \varphi_2 mkt_{t-1} + \beta_{t-1} (R_{m_t} - R_t) + \varphi_3 r_{t-1} (R_{m_t} - R_t) + ... \\
+ \varphi_4 mkt_{t-1} (R_{m_t} - R_t) + \varphi_5 VIX_{t-1} (R_{m_t} - R_t) + \beta_1 SMB_t + \beta_2 HML_t + \beta_3 gprod_t + \epsilon_t \tag{9}
\]

To estimate equation (9), we resort to a parsimonious approach, the OLS estimation. As
we can see, the conditional alpha and beta are easily identified using equations (4) and (5).
Equation (9) may be a very good approximation of the Kalman filter when using the
right explanatory variables. It might also be easily extended to account for measurement
or specification errors as explained in the appendix.

3. **Empirical results**

3.1 *The data*

This study is based on a sample of the indices of VanGreenwich (VG) strategies.
Statistical information on this sample appears in Table 1 and return spectra for selected

\(^{10}\) Market volatility is then another factor contributing to the revision of the time-varying alpha and beta from one period to the next.

\(^{11}\) In this situation, the squared market risk premium represents the co-asymmetry of a given portfolio return with the market return.
strategies in Figure 1. Our observation period of the monthly returns of these hedge fund indices runs from January 1995 to March 2010, for a total of 183 observations. The risk factors which appear in the F&F equation, - that is the market risk premium and the two mimicking portfolios: SMB and HML -, are for their part drawn from the French's website. The interest rate used to test the models is the US three months Treasury bill rate and the chosen market portfolio index is the S&P500. The period we analyze was plagued by four major financial crises: i) the Asian financial crisis (1997); ii) the Russian/LTCM crisis (1998); iii) the bursting of the high-tech market bubble (2000); iv) the 2007-2009 subprime market crisis (which is related to high risk mortgages). Our period of analysis is therefore rich in major stock market corrections. Despite these market collapses, Table 1 reveals that the VG hedge funds performed quite well during this period. The mean monthly return of these indices was 0.71% over this period, for an annual rate of 8.5%. This rate is higher than the annual mean return of the S&P500 over the same period, which amounted 5.5%. The low performers over this period were the short-sellers, the convertible arbitrage and macro strategies, whereas the high performers were the long-short, growth and market neutral strategies. Moreover, the standard deviation of returns differs greatly from one index to the next. The indices returns standard deviations are generally below the S&P500 one.

Several researchers argue that the strategies followed by hedge funds are similar to option-based strategies. And effectively, Table 1 reveals that some hedge fund strategies are actually similar to hedged option strategies, like the covered call and protective put ones. These option-based strategies have a beta which is quite low, of the order of 0.6 for at-the-money options, and may yet offer all the same quite high returns which approximate those

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12 The address of the French' website is: [http://mba.tuck.dartmouth.edu/pages_faculty/ken.french/data_library.html](http://mba.tuck.dartmouth.edu/pages_faculty/ken.french/data_library.html).
13 LTCM is the acronym of Long Term Capital Management, a highly levered US hedge fund which sustained massive losses in 1998.
shown in Table 1\textsuperscript{14}. The equity market neutral and the arbitrage strategies have a very low beta compared to other funds but their return exceed the overall mean return.

Furthermore, plain vanilla puts have a negative expected return. That might explain the low mean return of the short seller index over the period of analysis. At 0.18\%, it is well below the mean return of the whole set of strategies. Incidentally, the CAPM beta of the short seller index, equal to -1.01, is negative and quite high in absolute value over the period of analysis. According to the CAPM, the excess return of a portfolio having a negative beta should be low and even negative and that is the case of the short seller index.

Furthermore, according to Table 1, the composite index of hedge funds has less kurtosis than the market index given by the S&P500. However, this characteristic is not shared by some hedge fund strategy, the convertible arbitrage index having a kurtosis as high as 26.43. A high kurtosis means that rare or extreme events are more frequent than in a normal distribution, which suggests that payoffs are very nonlinear. Once more, we may relate these statistics to those associated to the cash-flows of option-based strategies. They have a relatively low standard deviation but a high degree of kurtosis in comparison with the returns of the market index which is priced in their returns.

3.2 Stylized facts on hedge fund returns spectra

Figure 1 provides the spectra of selected macroeconomic variables, factors and hedge fund strategies. The spectra of the two macroeconomic variables, the industrial production growth and the unemployment rate, have their usual plot. Industrial production growth displays high volatility at high frequencies while the unemployment rate, being in level, has

\textsuperscript{14} For a discussion of the beta and the mean return of option-based strategies, see: Whaley (2006), chapter 10.
the most of its volatility at very low frequencies. These plots are in conformity with the seminal analysis of Stock and Watson (1990)\textsuperscript{15}.

Figure 1 also present the spectra of factors entering in the Fama and French equation. Similarly to the analysis of Stock and Watson (1990), the spectrum of the most important variable explaining hedge fund returns, i.e. the market risk premium, suggests that the market risk premium is procyclical. It thus has a predictable component. But according to its spectrum, the $SMB$ variable is much more cyclical. This may be explained by the composition of this variable which is weighted by small and big firms. However, the $HML$ variable has a shorter cycle than the $SMB$ one. The difference between the spectra of these two last variables is thus interesting from an econometric point of view. Moreover, the spectrum of the $VIX$ has dampened fluctuations, which asymptotically vanishes.

More importantly, figure 1 suggest that the weighted composite hedge fund return index and the returns of the three selected strategies - growth, value index and short sellers - display high volatility at business cycle frequencies, which support our procyclical framework to analyze hedge fund returns. Note that the short sellers and growth strategies show also a high volatility at high frequencies.

3.3 Results

3.3.1 Kalman filter results

In this section, we consider in detail the estimation of equation (1) using the Kalman filter. The specification of the time-varying alpha and beta are given by equations (4) and (5).

\[\text{Insert Table 2 here}\]

\textsuperscript{15}See also Racicot (2010) for a review of spectral analysis techniques applied to financial data.
The Kalman filter estimation of this system of equations, which is reported in Table 2, reveals that this system performs quite well. The overall performance of the model is the highest for the weighted composite index with the highest likelihood ratio ($L$) and the lowest Akaike information criterion ($AIC$). The $L$ statistic is also quite high and the $AIC$ is relatively low\footnote{To see the correspondence between the $R^2$ and the log likelihood statistic in table 1, let us notice, for instance, that the equation of the excess return of the equity hedge strategy has a $R^2$ of 0.83 when estimated by the OLS method and a log-likelihood statistic of -173 when estimated by the Kalman filter.} for three of the indices: market neutral, long-short and value index. The model performs moderately for the distressed securities and growth strategies. The model is less satisfying for the futures index.

In the estimation of the measurement equation given by equation (1), the market risk premium is the most important risk factor to impact on hedge fund returns, followed by the $SMB$ one. The final state conditional beta provided by the $sv2$ variable coefficient in Table 2 has a high of 1.10 for the growth strategy and a low of 0.1596 for the equity market neutral strategy. All these betas are generally under the market portfolio one, being 1 by definition, because hedge funds usually reduce their market exposure with hedging operations. Let us notice that the betas appearing in Table 2 are those corresponding to the final state of our estimation period. To analyse the dynamics of the conditional alphas and betas corresponding to the VG indices, we must build the state series of these coefficients.

Before considering this subject, note in Table 2 that the $SMB$ factor is significant at the 99% confidence level for all strategies analyzed except for the futures one for which it is not significant. According to Capocci and Hübner (2004), hedge funds prefer to hold stocks of small firms over those of bigger ones. The $HML$ factor is for its part significant at the 10% level for only three indices: the growth index, the long-short index, and the weighted composite one. The impact of this factor is generally lower than $SMB$ in most of the empirical studies on that matter.
One of our main contributions in this study is to analyze the procyclicality of risk-taking in the hedge fund industry. In that respect, we have introduced the smoothed growth of industrial production in the Fama and French model. This variable is significant at the 95% confidence level for the weighted composite, which stands for the representative hedge fund, and for the value index. Consistent with the economic theory, the sign of this variable is negative. Indeed, in expansions, the risk premium decreases, investors being less averse to risk following the improvement of business conditions, and the economic theory predicts that expected returns should then be lower, expected returns being countercyclical because the risk premium is itself countercyclical (Cochrane 2005).

Let us now consider the states series of the filtered conditional alpha and beta. As there is a training period for the Kalman filter, there are usually spikes at the start of the plots of the state beta. We will begin our discussion with the conditional beta state series which is a priori more manageable than the conditional alpha state variable.

The final state conditional beta (sv2), associated to the market risk premium is very significant for every index as indicated by the p-values of this coefficient in Table 2. It is equal to 0.48 for the weighted composite index, the beta of hedge funds being usually quite moderate relatively to traditional stock portfolios like the mutual funds ones. The indices having the largest beta are the growth index and the futures one. These betas are larger than one. The indices having the lowest beta are the market neutral, as expected, and the distressed securities index.

Regarding the conditioning variables, we note at Table 2 that the interest rate ($r_f$) has a negative impact on hedge funds betas, which support our hypothesis since an increase in interest rate signal a market deterioration, which lead hedge funds to take less risk. In other respects, according to the condition market variable ($mkt_rf$), hedge funds take more risk when the market returns, as measured by the S&P500 index, increases, but this effect is
quite low and not significant for the distressed securities and market neutral strategies. Finally, financial market volatility as measured by VIX impact positively and significantly on the market returns of all strategies except the value index one, for which the exposure to volatility is negative and insignificant. In light of our previous comments about the sign of this variable, hedge funds seems conditioned by the payoffs related to forward market volatility since the value of an option is dominated by its volatility.

The plots of the betas indicate that it is far from being constant and has a procyclical behaviour. In Figure 2, we compare the profile of the state beta of the weighted composite index to the unemployment rate, a well-known countercyclical time series. We note that the state beta is negatively correlated to the unemployment rate. Moreover, the beta has an inverse cycle compared to unemployment: it is thus procyclical. However, remind that the unemployment rate is a lagged indicator of economic activity while the beta, being the result of a investment decision process, is forward looking. In that respect, the behaviour of the beta during the 2007-2009 subprime crisis is very interesting. During the first-half of the crisis, the unemployment rate jumps significantly while the beta drops substantially, which suggest that the representative hedge fund reduced greatly its level of market risk. But being a lagged indicator, the unemployment rate continued to increase in the last phase of the crisis. However, the beta resumed its increase, in expectation of the upcoming expansion. In this sense, the beta is forward-looking.

As shown in Figure 3, the conditional beta of the weighted composite index decreased during the 1997 Asian crisis before resuming its increase in 1998. Thereafter, following the first recession of the millennium, the beta decreased from the beginning of 2000 till the end of 2002, which paved the way to a market recovery. It almost doubled from 2003 to the
middle of 2005. It decreased progressively thereafter in expectation of an economic slowdown and in reaction to the corporate accounting scandals. The conditional alpha related to the weighted composite index has a profile similar to the beta but is more volatile (Figure 3). The conditional alpha associated to the weighted composite decreases after the Asian crisis, gaining momentum during the technological bubble. During this episode, the estimated alpha decreased from a high of 1% monthly to a low close to 0%, which suggests that the alpha puzzle is perhaps not a puzzle when we account for the cyclical behaviour of alpha. The alpha puzzle could thus be an artifact of the static framework used by the empirical studies on the alpha puzzle. Our procyclical approach to the alpha seems thus be more relevant to study the alpha process. As in the case of beta, the profile of the alpha is particularly interesting during the 2007-2009 subprime crisis. According to Figure 3, it decreases to a low of 0% in the middle of the crisis before recovering thereafter, a profile similar to the beta. In summary, the level of the alpha is conditioned by the level of risk taken by a hedge fund, as measured by the beta. Figure 3 indicates that when the beta increases, the alpha increases and vice-versa when the beta decreases, in line with the classical risk-return trade-off.

3.3.2 Robustness checks: Kalman filter and conditional models

As robustness checks, we compare in this section the cycle of state and conditional beta and alpha. Figure 4 provides this comparison for the VG weighted composite index. We note that the conditional beta (equation (5)) has a procyclical profile which is very similar to the one of the state beta, which supports the Kalman filter estimation. Both beta are very synchronized during an economic slowdown, like during the technological bubble burst and the subprime crisis. But when the economy recovers, we note that the state beta is
forward looking in regard to the conditional one. For example, during the 2003-2007 economic recovery, the increase of the conditional beta lagged the increase in the state beta by many months. The same phenomenon is observed during the last period of the subprime crisis, whereas the state beta jumps while the conditional beta increases only modestly. Note also that the state beta began its decrease before the subprime crisis, which is not the case for the conditional beta. The Kalman filter thus seems better appropriate to model the forward looking properties of time-varying betas.

Figure 5, which plots the Kalman filtered and conditional alphas for the VG weighted composite index, sheds more light on the alpha puzzle in a dynamic setting. It seems that the alpha puzzle is much less problematic in our procyclical framework to analyze hedge fund returns. Contrary to the previous empirical analyses of hedge fund returns, the alpha may become nil or even negative in periods of economic contraction. This pattern is more pronounced for the Kalman filtered series than the one obtained by the conditional model. It is thus interesting to note that the alpha is positive in economic expansion, even moving on an upward trend in economic expansion. Note that in the 1990s, there seems to be an alpha puzzle but it is seems related to the long period of economic expansion which prevails then. Therefore, studies on hedge fund returns during this period might have observed a false alpha puzzle being an artefact of their analysis conducted in a static framework. Note also in Figure 5 that the cyclical behaviour of the alpha is asymmetric, having a tendency to decrease more in contractions than it increases in expansions. Even if our sample is not long enough to be conclusive on that, the alpha seems mean-reverting towards a mean which declines slowly.

Insert Figures 6 and 7 here
The cyclical properties of the weighted index return are shared by the returns of the specific strategies. We note in Figures 6 and 7 that their betas display procyclicality, a counterfactual result in regard of the nature of these strategies. In line with the weighted composite index, the betas of both strategies returned on an upward trend well before the end of the the 2007-2009 subprime crisis, the swing being more pronounced with the Kalman filter. Moreover, the conditional beta seems more sensitive to market conditions than the Kalman filtered one, especially for the market neutral strategy.

Insert Figures 8 and 9 here

It is also interesting to study the procyclical behaviour of two strategies having bigger betas than the hedge funds, which like the market neutral and the long-short strategies, have a low market exposure. Figures 8 and 9 provide the cyclical behaviour of the value index and the growth index whose betas are respectively 0.56 and 0.76. We note that the procyclical behaviour of these two strategies is quite different. The value index has a procyclical profile which is similar to the weighted composite index and is quite sensitive to the market conditions. Moreover, as the weighted composite, the value index is forward-looking, especially before and during the 2007-2009 subprime crisis. However, contrary to the weighted composite index, note that the Value index conditional beta is more forward-looking than the Kalman filtered beta, which suggests that the Value index strategy is very sensitive to financial market conditions. Another interesting aspect of the cyclical profile of the growth index is that during the expansion 2002-2007, the growth index beta, both computed by the Kalman filter and the conditional model, began to decrease well before the end of the expansion after having peaked at a level near 1, which is not the case for the value index which follows economic conditions more tightly.

Insert Figures 10 and 11 here
Finally, we consider two very specialized strategies: the futures and the distressed securities ones (Figures 10 and 11). Because of the instability of their returns, it was difficult to apply the Kalman filter so we resorted only to the conditional model. The procyclical profile of the futures index is close to the one given by the weighted composite index. However, the fluctuations of the futures index are much wider, the beta fluctuations being in a range of $[-0.5, 0.4]$ in Figure 10. This profile is also very different from the ones of the long-short and market neutral strategies, which also explains why the Kalman filter has difficulties to capture the time profile of the futures index. In other respects, even if it is quite less volatile than the futures index, one interesting dimension of the distressed index is that, contrary to the other indices, the distressed beta increases during contractions. Actually, distressed securities are concentrated in contraction periods so distressed funds take more risk during economic slowdowns.

4. Conclusion

In this paper, we revisit hedge fund return models relying on a procyclical setting based on two dynamic procedures: the Kalman filter and the conditional model approach (Racicot and Théoret, 2007, 2008). This procedure appears more rigorous than the usual least squares procedure usually used to compute the conditional alpha and beta. In the Kalman filter approach, the model coefficients are updated each period in an optimization framework taking into account all information accumulated till this period. We also consider the conditional model as an approximation to the Kalman filter.

Since we resort to a procyclical approach to study hedge fund returns in this paper, we introduce cyclical variables in our hedge fund returns models, like the industrial production, unemployment rate, the rate of vacancy and the VIX, among others. We find that these variables capture quite well the cyclical behaviour of the hedge fund risk-return
trade-off. To the best of our knowledge, our study is the first to consider the cyclical profile of hedge fund returns in detail.

Our study reveals that the state and conditional alphas are quite responsive to the business cycle, usually increasing during expansions and decreasing during contractions. The Kalman filter and the conditional model both support this procyclical profile. We may thus conclude that the absolute hedge fund return, as measured by the alpha of Jensen, has a cyclical behaviour, a new aspect for this performance measure not present in the literature. We also noted that the alpha had a tendency to mean-revert from one business cycle to the other. In order to check for its robustness, this result needs further investigation.

The time-varying beta also displays a pronounced procyclical profile for both the Kalman filter and the conditional model. The conditional beta responds positively to the market risk premium and negatively to the level of interest rate, a quite rational behaviour. Moreover, the betas are conditioned by a cycle which may be explained quite easily by the profile of the macroeconomic variables we use. Incidentally, the performance of models integrating conditioning information is usually much better to explain the expected excess returns of the hedge funds because the conditional beta is significantly related to macroeconomic information. A promising research avenue is to consider the impact of procyclicality of hedge fund risk on the business cycle. Actually, do hedge funds exacerbate the amplitude of the business cycles, a very important question from the point of view of macroprudential policy.

Finally, we propose, in the appendix of this article, a possible framework to account for the biases that might be caused by errors-in-variables in the setting of the Kalman Filter (Racicot and Théoret 2010a). This approach might be justified by the fact that financial variables might be contaminated by measurement errors, thus causing a bias in the estimation process. Several authors have worked on the subject (e.g. Racicot & Théoret,
2008a, b and 2010b) and there is a strong evidence for this problem. We thus think that it is a good idea to do further experiments on how to supplement the Kalman Filter with another algorithm to account for measurement errors.
References


Appendix

Errors-in-Variables and the Kalman Filter

Another aspect of the modeling of hedge fund returns which is relatively neglected in the empirical literature is the behaviour of the Kalman Filter in the context of errors in the explanatory variables. Here our goal is to propose a theoretical model for handling this kind of problems (errors-in-variables and other kinds of specification errors) but we leave the interesting subject of how to empirically implement this new approach to further research.

As shown in Coën & Racicot (2007) and Racicot & Théoret (2008a, 2009), it is quite easy theoretically speaking to account for errors in the explanatory variables. One must transform a regression model into an artificial Hausman regression one. In our particular case, our basic model could be extended to account for the problem of errors-in-variables as

\[ r_{pt} - r_{ft} = \alpha_t + \beta_t (r_{mt} - r_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \phi_1 \tilde{E}_t + \phi_2 \tilde{E}_2 + \phi_3 \tilde{E}_3 + e_t \]  \tag{10}

where the time-varying parameters could be defined initially, for parsimony, in their simplest form, which is a standard random walk process

\[ \alpha_t = \alpha_{t-1} + v_{it} \]  \tag{11}

\[ \beta_t = \beta_{t-1} + v_{it} \]  \tag{12}

In equation (10) the variables \( \tilde{E}_i \), \( i=1,...,3 \), are included to account for the potential correlation between the explanatory variables and the innovation \( e_t \). We compute these elements as follows

\[ \hat{E} = x - \hat{X} \]  \tag{13}

where \( x \) is the matrix of the explanatory variables expressed in deviation from its mean, and

\[ \tilde{E} = Z(Z'Z)^{-1}Z'x \]  \tag{14}
where $Z$ is a matrix of instrumental variables which use as instruments the cumulants or the higher moments of order two and three of the explanatory variables. To operationalize the computation of this procedure, we use a computer code which runs in EViews and developed by the authors (Racicot and Théoret 2008a, b). As explained previously, the time-varying coefficients can also be explained by some factors. This means that the basic random walk model, which constrains the time-varying coefficients to behave in a purely recursive fashion, can be extended further to include some explanatory components, as shown in equations (4) and (5). We leave these and other potential generalizations of that model for future work.
### Table 1 Descriptive statistics of the Van Greenwich indices returns, 1995-2010*

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>sd</th>
<th>Skew</th>
<th>Kurtosis</th>
<th>CAPM-beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distressed securities</td>
<td>0.68</td>
<td>1.04</td>
<td>4.79</td>
<td>-7.44</td>
<td>2.06</td>
<td>-1.47</td>
<td>6.86</td>
<td>0.22</td>
</tr>
<tr>
<td>Equity Market Neutral</td>
<td>0.87</td>
<td>0.80</td>
<td>8.10</td>
<td>-2.53</td>
<td>1.41</td>
<td>1.33</td>
<td>8.95</td>
<td>0.08</td>
</tr>
<tr>
<td>Futures</td>
<td>0.67</td>
<td>0.21</td>
<td>7.71</td>
<td>-6.80</td>
<td>3.10</td>
<td>0.18</td>
<td>2.72</td>
<td>-0.08</td>
</tr>
<tr>
<td>Macro Index</td>
<td>0.55</td>
<td>0.66</td>
<td>4.00</td>
<td>-2.95</td>
<td>1.45</td>
<td>0.29</td>
<td>3.06</td>
<td>0.27</td>
</tr>
<tr>
<td>Market neutral group</td>
<td>0.93</td>
<td>0.92</td>
<td>7.20</td>
<td>-6.06</td>
<td>1.48</td>
<td>-0.61</td>
<td>8.99</td>
<td>0.20</td>
</tr>
<tr>
<td>Short sellers</td>
<td>0.18</td>
<td>-0.10</td>
<td>11.41</td>
<td>-6.88</td>
<td>3.61</td>
<td>0.56</td>
<td>3.46</td>
<td>-1.01</td>
</tr>
<tr>
<td>Value index</td>
<td>0.61</td>
<td>1.11</td>
<td>5.68</td>
<td>-9.65</td>
<td>2.54</td>
<td>-2.12</td>
<td>5.94</td>
<td>0.56</td>
</tr>
<tr>
<td>Arbitrage Index</td>
<td>0.87</td>
<td>0.90</td>
<td>4.10</td>
<td>-8.58</td>
<td>1.38</td>
<td>-2.40</td>
<td>17.99</td>
<td>0.16</td>
</tr>
<tr>
<td>Convertible Arbitrage Index</td>
<td>0.31</td>
<td>0.60</td>
<td>6.55</td>
<td>-19.31</td>
<td>2.98</td>
<td>-3.78</td>
<td>26.43</td>
<td>0.40</td>
</tr>
<tr>
<td>Growth index</td>
<td>1.04</td>
<td>1.19</td>
<td>20.10</td>
<td>-12.99</td>
<td>4.53</td>
<td>0.43</td>
<td>5.50</td>
<td>0.76</td>
</tr>
<tr>
<td>Long-Short</td>
<td>1.09</td>
<td>1.31</td>
<td>13.20</td>
<td>-9.24</td>
<td>3.02</td>
<td>0.04</td>
<td>5.20</td>
<td>0.52</td>
</tr>
<tr>
<td>Mean of indices</td>
<td>0.71</td>
<td>0.78</td>
<td>8.44</td>
<td>-8.40</td>
<td>2.51</td>
<td>-0.60</td>
<td>8.65</td>
<td>0.19</td>
</tr>
<tr>
<td>Weighted composite</td>
<td>0.56</td>
<td>0.90</td>
<td>4.75</td>
<td>-5.96</td>
<td>1.86</td>
<td>-1.01</td>
<td>5.18</td>
<td>0.37</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.46</td>
<td>1.29</td>
<td>11.06</td>
<td>-18.47</td>
<td>4.62</td>
<td>-1.13</td>
<td>5.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* The statistics appearing in this table are computed on the monthly returns of the Van Greenwich (VG) indices over the period running from January 1995 to March 2010. The weighted composite index is computed over the whole set of the VG indices. The CAPM beta is estimated using the simple market model, that is: \( R_i - R_f = \alpha + \beta (R_{m} - R_f) + \epsilon_i \), where \( R_i \) is the return of the index \( i \), \( R_m \) is the S&P500 return, \( R_f \) is the riskless rate and \( \epsilon_i \) is the innovation.
Table 2 Kalman filter estimation*

| Model          | conditional alpha | conditional beta | | | | |
|----------------|------------------|------------------|---|---|---|---|---|---|---|
|                | sv1   | rf(-1) | mkt_rf(-1) | sv2   | rf(-1) | mkt_rf | VIX | smb  | hml  | prod. ind. | L   | AIC  |
| Distressed securities | 0.3097 | -0.0097 | -0.0068 | 0.3593 | -0.0014 | 0.0000 | 0.0043 | 0.1689 | 0.0403 | -0.0121 | -344.64 | 3.86 |
| Market Neutral | -0.1904 | -0.0493 | -0.0165 | 0.1596 | -0.0145 | 0.0001 | 0.0027 | 0.1299 | -0.0022 | -0.0183 | -295.16 | 3.32 |
| Long-Short | -0.0571 | -0.0525 | 0.0225 | 0.6152 | -0.0150 | 0.0001 | 0.0041 | 0.2618 | -0.0669 | -0.0563 | -296.72 | 3.34 |
| Value Index | -0.0367 | -0.0560 | 0.0193 | 0.3985 | 0.0072 | 0.0025 | -0.0002 | 0.2815 | 0.0364 | -0.0861 | -300.67 | 3.38 |
| Growth Index | -0.4483 | -0.0700 | 0.0314 | 1.1032 | -0.0591 | 0.0055 | 0.0007 | 0.3416 | -0.2984 | -0.0472 | -372.19 | 4.17 |
| Futures Index | 0.1807 | 0.0145 | -0.0134 | 1.0342 | -0.1540 | 0.0108 | 0.0019 | 0.0041 | 0.0596 | 0.1132 | -503.46 | 5.60 |
| Weighted composite | 0.0292 | -0.0229 | 0.0125 | 0.4782 | -0.0216 | 0.0040 | 0.0002 | 0.1995 | -0.0424 | -0.0449 | -260.06 | 2.94 |

* Model 1 is made up of the measurement equation (1) and transition equations (2) and (3). These equations are estimated simultaneously resorting to the Kalman filter. For each strategy, the first line of numbers is the estimated coefficients of the variables located at the head of the columns and the second line gives the corresponding t-statistics (in italics). The L statistic is the log likelihood associated to an estimation. The sv1 coefficient is the final state of the conditional alpha and the sv2 coefficient is the final state of the conditional beta.
Figures

Figure 1

Macroeconomic variables spectra

Factor spectra
Hedge Fund Returns spectra

Figure 2 US unemployment rate and VG hedge fund composite index

Note: US recessions are shaded.
Figure 3 State series for the alpha and beta of the VG weighted composite index

Note: US recessions are shaded.

Figure 4 State series and conditional model for the alpha of the VG weighted composite index

Note: US recessions are shaded.

Figure 5 State and conditional series for the alpha of the VG weighted composite index

Note: US recessions are shaded.
Figure 6 State and conditional beta of the equity market neutral strategy

Figure 7 State and conditional beta of the long-short strategy

Figure 8 State and conditional beta of the value index strategy

Note: US recessions are shaded.
Figure 9 State and conditional beta of the growth index strategy

Note: US recessions are shaded.

Figure 10 State and conditional beta of the futures index strategy

Note: US recessions are shaded.

Figure 11 Conditional series for the beta of the distressed securities index

Note: US recessions are shaded.