Monitoring portfolio tail risk over the business cycle: Evidence from hedge fund strategies

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Abstract

We study how downside risk taken by hedge fund strategies responds to macroeconomic and financial shocks. We find that the impact of shocks on strategy multi-moment risk displays an asymmetric behavior, being much more significant and nonlinear during the subprime crisis than during the recent expansion period. However, our results indicate that managers of hedge fund strategies seek to monitor their tail risk during crises—i.e., that their portfolios seem to behave as puts, which are kurtosis reducers in times of turmoil. However, while some strategies (e.g., futures) succeed quite well in controlling their tail risk during crises, others (e.g., equity market neutral) have difficulties to do so. This is an important result for institutional investors who believe that hedge funds are “hedged” and thus immunized against adverse macroeconomic shocks—especially, market shocks.

Keywords: Hedge fund; Tail risk; Local projection; Business cycle; Systemic risk.

JEL classification: C13; C58; G11; G23.

Suivi du risque extrême de portefeuilles sur le cycle économique : Le cas des stratégies des fonds de couverture

Résumé

Nous analysons comment le risque de perte supporté par les stratégies de fonds de couverture réagit aux chocs macroéconomiques et financiers. Nous trouvons que l’impact des chocs sur le risque associé aux moments des rendements des fonds fait montrer d’un comportement asymétrique, ayant été plus significatif et non-linéaire durant la crise des subprimes que durant la période récente d’expansion économique. Cependant, nous résultats donnent à penser que les gestionnaires des stratégies des fonds de couverture cherchent à montrer leur risque extrême durant les crises—c.-à-d. que leurs portefeuilles semblent se comporter comme des options de vente, agissant des réducteurs de leptokurtisme en temps d’agitation financière. Cependant, alors que certaines stratégies (e.g., gestionnaires de contrats à terme) réussissent assez bien à contrôler leur risque extrême durant les crises, d’autres (e.g., risque-neutre en actions) éprouvent des difficultés de suivi. Ces constats représentent un résultat important pour les investisseurs institutionnels qui croient que les fonds de couverture sont couverts et par conséquent immunisés contre les chocs macroéconomiques adverses, et plus spécialement les chocs de marché.

Mots-clés : Fonds de couverture; Risque extrême; Projection locale; Cycle économique; Risque systémique.

Classification JEL : C13; C58; G11; G23.
1. Introduction

Prudent investors dislike downside risk (Geiss et al., 1980; Eeckhoudt et al., 2005). However, it is well-known that the standard deviation of stock returns or a stock beta does not adequately measure this kind of risk. To really account for downside risk, we rely on measures of tail risk. Moreover, the indicators of financial risk ought to be analyzed in a dynamic setting. Indeed, risk-averse investors dislike a transfer of risk (or probability mass) from wealthy or good states—e.g., economic expansions—to poorer or bad states—e.g., crises or recessions. These transfers are the foundation of stochastic dominance theory (Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970, 1971). This is an obvious source of asymmetry in the risk preferences of investors according to the phase of the business cycle.

Institutional investors—like pension funds and university endowments—are particularly concerned about tail risk in the advent of an adverse macroeconomic event. By loose arguments to the effect that they are somehow “hedged” and adopt market neutral strategies, hedge funds were made attractive to institutional investors who were concerned by such a possibility. Hedge funds could act as portfolio insurers during crises if their payoffs are similar to put options. As shown by Bali et al. (2017), puts behave as kurtosis reducers during crises like the Lehman Brothers bankruptcy in September 2008. This would be the case if hedge funds are tracking tail risk. As shown by Mitchel and Pulvino (2001), the returns of many hedge fund strategies usually behave as portfolios of short puts. During crises, if they act as portfolio insurers, managers of these strategies should make the payoffs of their portfolios similar to those of long puts (Agarwal et al., 2017b). Actually, trailing tail risk is ideally associated with an increase in skewness and a reduction in kurtosis during crises, a result difficult to obtain according to the Wilkins’ lower bound (1944). Actually, in a risk neutral universe, the kurtosis of the distribution of put returns decreases during a crisis but its skewness is also reduced (Bali et al., 2017).

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1 i.e., co-skewness and co-kurtosis.
2 i.e., risk-averse investors prefer a smooth consumption plan over time (Cochrane, 2005).
3 Even if the framework of Mitchel and Pulvino is a step forward towards a dynamic analysis since it embeds options, it is not truly dynamic in a Kalman filter / Bellman sense. The same argument could be transposed to the articles published by Fung and Hsieh (1997, 2001, 2002, 2004).
al., 2017). However, kurtosis weights much more in the utility function of an investor during crises—i.e., when a high kurtosis could then lead to a substantial negative payout.

In this study, we analyze how hedge funds trail their risk—especially tail risk—over the business cycle. More precisely, we aim at examining how the managers of hedge fund strategies monitor their downside risk over the cycle to satisfy the preferences of their investors. To do so, we rely on a non-linear local projection method (Jordà 2004, 2005) for analyzing the response of multi-moment risk of hedge fund strategies—as measured by beta, co-skewness and co-kurtosis—to macroeconomic and financial shocks. We perform this local projection first on a recent expansion period—i.e., from 2013 to 2016—and then over the subprime crisis to decrypt the asymmetric pattern of hedge fund behavior towards risk in expansion and recession (or crisis). Indeed, previous studies have found many asymmetries (nonlinearities) in the behavior of financial institutions dependent on the stance of the business cycle (e.g., Jawadi and Khanniche, 2012; Bali et al., 2014; Calmès and Théoret, 2014; Namvar et al., 2016; Racicot and Théoret, 2016; et al., 2017; Agarwal et al., 2017b).

Similarly to Bali et al. (2014) and Racicot and Théoret (2016) who focus on the impact of macroeconomic uncertainty on hedge fund returns and betas, our main contribution is to show that this kind of uncertainty has also an important role to play in the dynamics of the risk associated with the higher moments of strategies’ returns—i.e., co-skewness and co-kurtosis. More precisely, hedge funds tend to track beta and higher moment risk either to hedge it (reduce it)—especially in recession—or to capture positive payoffs associated with risk premia—particularly in expansion. Our study complements the recent paper of Agarwal et al. (2017b) who find that hedge funds appear to monitor a measure of tail risk these authors developed. They show that hedge funds tend to sell insurance in good times but adopt the opposite behavior in times of turmoil by buying protection to reduce their tail risk4. However, Agarwal et al. (2017b) are mainly interested in the microstructure of tail-risk timing in the hedge fund industry while our paper focuses more on the macroeconomic dynamics of hedge fund risk5.

We find that the response of hedge funds is actually asymmetric dependent on the stance of the business cycle—i.e., recession or expansion. During recessions, the reaction of hedge fund risk to shocks, regardless of their provenance, is less smooth, stronger and more significant than during expansion. More importantly, we find that risk is more difficult to monitor or hedge in the subprime crisis than in an expansion period. During crises, hedge funds may lose temporarily the control over their risk measures, especially following VIX or credit

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4 i.e., the payoffs of many hedge fund strategies are similar to those of a short put in expansion and to those of a (long) put in recession. There is thus an obvious asymmetry or change in regime regarding their behavior. See also Agarwal et al. (2004).

5 Note that Agarwal et al. (2017b) have correlated their tail risk measure with the global uncertainty indicator developed by Bali et al. (2014), which is the first principal component of well-known macroeconomic and financial uncertainty indicators. They find a negative correlation. Although the authors do not discuss this result, it may be further evidence that hedge funds do track their tail risk.
shocks to which they are particularly exposed (Gregoriou, 2004). The co-skewness of hedge fund strategies is markedly difficult to track during a crisis. However, some strategies succeed quite well in fine-tuning their risk during a crisis. This is particularly the case for the growth strategy—which displays hedging properties concerning market reversals (Campbell et al., 2010)—and for futures\(^6\) or short-sellers strategies owing, in large part, to their short sales.

In line with Agarwal et al. (2017a) who find that volatility of volatility plays a key role in the explanation of hedge fund cross-sectional returns, one of our contributions is to show that nonlinearities, and especially those related to \(\text{VIX}\), are crucial for the dynamics of our nonlinear local projection model. In this respect, a comparison of the results obtained with linear and nonlinear impulse response functions (IRF) indicates that the squared endogenous variables—e.g., \(\text{VIX}^2\)—are usually much more important than their levels, particularly during crises.

Our last contribution is to shed more light on the asymmetries related to systemic risk in the hedge fund industry using an underpinning developed by Beaudry et al. (2001) and transposed to the analysis of systemic risk in the financial sector by Baum et al. (2002, 2004, 2009), Calmès and Tchéret (2014), and Racicot and Tchéret (2016). We find that tail risk may be an important source of systemic risk for hedge funds during crises. Therefore, monitoring only the beta may lead to a serious underestimation of risk borne by hedge fund strategies during crises\(^7\).

This paper is organized as follows. Section 2 presents the time-varying risk measures used in this paper—i.e., beta, co-skewness and co-kurtosis—as well as the theoretical and empirical background, particularly the nonlinear local projection method we use to implement our experiments. Section 3 presents the database and the stylized facts. In Section 4, we analyze the impulse response functions of our risk measures to shocks computed with a nonlinear local projection model (Jordà, 2005) over the 2013-2016 expansion period and over the subprime crisis. We also compare these nonlinear impulse response functions to standard linear ones. Section 5 focuses on systemic risk in the hedge fund industry. Section 6 provides a robustness check related to the asymmetric responses of our measures of hedge fund risk to shocks while Section 7 concludes.

2. Methodology

2.1 The empirical measures of hedge fund conditional (time-varying) return moments

\(^4\) This result is consistent with Asness et al. (2001) who find that the managed futures strategy displays market-timing skills.

\(^7\) In this respect, Agarwal and Naik (2004) argue that the mean-variance approach may underestimate portfolio losses and that this underestimation may be substantial for portfolios with low volatility. In our study, we find that the equity market neutral strategy—which displays low volatility—may be quite risky during crises when we account for higher moment risk. It is not so neutral after all (Duarte et al., 2007; Patton, 2009). See also Agarwal et al. (2017b).
Similarly to the beta which is the keystone of the two-moment CAPM, the concepts of co-skewness and co-kurtosis originate from the four-moment CAPM, which takes the following form (Fang and Lai, 1997):

\[
E(R_i) - r_i = \beta_i Cov(R_m, R_i) + \beta_i^2 Cov(R_m^2, R_i) + \beta_i Cov(R_m^3, R_i)
\]

where \(E(R_i)\) is the expected value of return \(i\); \(r_i\) is the risk-free rate and \(Cov(.)\) is the operator of covariance. The unscaled beta, co-skewness and co-kurtosis of the asset which is priced are defined as \(Cov(R_m, R_i)\), \(Cov(R_m^2, R_i)\), and \(Cov(R_m^3, R_i)\), respectively. As usual, the risk associated with \(R_i\) is thus seen as co-movements between this return and the stock market return (unscaled beta), its square (unscaled co-skewness) and its cube (unscaled co-kurtosis), respectively.

To arrive at relative measures of risk, we scale covariances in Eq. (1). The well-known definition of the beta of asset \(i\) is:

\[
\beta_i = \frac{Cov(R_m, R_i)}{Var(R_m)}
\]

which is the scaling of the first \(Cov(.)\) term on the RHS of Eq.(1). Analogously to the CAPM, we also scale the two other covariance expressions in this equation, which give rise to co-skewness and co-kurtosis, respectively:

\[
co-skewness_i = \frac{Cov(R_m^2, R_i)}{[Var(R_m)]^{3/2}}
\]

\[
co-kurtosis_i = \frac{Cov(R_m^3, R_i)}{[Var(R_m)]^3}
\]

Similarly to the beta which measures systematic market risk, co-skewness and co-kurtosis are two dimensions of systematic tail risk. To make our three measures of risk time-varying, we rely on the multivariate GARCH (MGARCH, Bollerslev et al., 1988; Engle, 2002). The simple system used to implement this procedure writes as follows:

\[
f(y_i, \beta) = \varepsilon_i
\]

In Eq. (5), \(y\) is a vector of endogenous variables and \(\varepsilon\) is a vector of possibly serially correlated disturbances. Each equation of the system has its simplest expression: \(y = \text{constant} + \varepsilon\). In our framework, the vector \(y\) takes the form: \(y_i = [R_m, R_m^2, R_m^3, R_m^4]\). The aim of the estimation is to find an estimate of the vector of parameters \(\beta\) which is then used to compute the conditional covariance and variance measures appearing in equations (2), (3) and (4), respectively. In this study, we adopt the dynamic conditional correlation algorithm using the
Engle (2002) procedure to estimate the $\beta$ vector since it is a parsimonious approach in terms of the number of parameters to estimate.

2.2 The theoretical model

Since we adopt a vector autoregressive (VAR) approach, our model must be parsimonious with respect to the number of explanatory variables. Indeed, given that each variable is lagged over several periods, this quickly reduces the number of degree of freedom. The main objective of our paper is to study the response of hedge fund strategies’ moments—i.e., beta, co-skewness and co-kurtosis—to macroeconomic and financial shocks. To conduct our experiments, we thus borrow from the empirical APT model of Chen et al. (1986), and from the multifactorial model proposed by Gregoriou (2004) to study the market timing of hedge funds:

$$y_t = \lambda_0 + \lambda_{\text{output gap}} + \lambda_{\text{credit spread}} + \lambda_{\text{term spread}} + \lambda_{\text{VIX}} + \varepsilon_t$$

where $y_t$ are our measures of multi-moment risk—i.e., beta, co-skewness and co-kurtosis; output_gap is computed using U.S. quarterly GDP; credit_spread is the spread between BBB and AAA corporate bond yields; term_spread is the spread between the U.S. Government ten-year interest rate and the three-month Treasury bills rate, and VIX measures the volatility of the U.S. stock market. In our model, we have substituted the VIX for the inflation rate which was a component of the original Chen et al. (1986) model. Note that the choice of the term spread, the credit spread and the industrial production growth rate—here replaced by the output gap, which is more stable than the industrial production growth—is supported by many researchers in the field of hedge funds (Kat and Miffre, 2002; Amenc et al., 2003; Brealy and Kaplanis, 2010; Bali et al., 2014; Lambert and Platania, 2016). Furthermore, Agarwal et al. (2017a) and Lambert and Platania (2016) find that the implied volatility (VIX) of the U.S. stock market is an important driver of hedge fund returns and hedge fund exposures to the stock market and Fama and French factors. Finally, according to Gregoriou (2004), the model we use is justified by the fact that hedge funds trade options (VIX), and have significant exposure to credit risk (credit_spread) and term risk (term_spread). In addition to these variables, we add the output gap in order to study the procyclicality of the risk borne by hedge fund strategies.

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8 For more detail on the computation of the linear VAR with hedge fund applications, see Racicot and Théoret (2017).

9 To compute the quarterly output gap, we first take the log of real GDP. We then detrend this transformed series with the Hodrick-Prescott filter using a smoothing coefficient ($\lambda$) equal to 1600—the trend of the series being a measure of potential output. The resulting residuals are the output gap measure.

10 The reasons for this substitution are the following. First, because over our sample period, inflation is not a matter of concern as it was when Chen et al. (1986) conducted their study. Second, given its high level of volatility, the significance of the inflation rate was usually quite weak in the Chen et al. (1986) estimations or at least lower than the other included explanatory variables. Third, since we run our model with VARs, its formulation ought to be parsimonious. Fourth, the VIX is one of the most important variables explaining hedge fund returns since these funds are much involved in structured products.
It seems reasonable to assume that hedge funds decrease their exposure to risk in bad states—i.e., recession or crisis—and increase it in good states—i.e., expansion. Hedge fund portfolio managers should aim at decreasing their beta and co-kurtosis as well as increasing their co-skewness in bad states and reversing the signs of these exposures in good states. In our canonical model given by equation (6), bad states are obviously associated with a decrease in the output gap and a rise in the credit spread, which lead to an increase in investors’ risk aversion and a rise in firms’ bankruptcies. Moreover, an increase in the term spread should also entail a decrease in hedge fund risk taking, since we may relate a rise in this spread to an increase in the liquidity premium (Lambert and Plantania, 2016). The slope of the yield curve is particularly steep in recession. Indeed, the liquidity constraint of the economic agents is then binding and the liquidity premium increases for long-term investments (Gollier, 2001). Finally, an increase in the VIX also co-moves with bad states owing to the Black (1976) leverage effect, whereby an increase in the VIX is associated with a drop in the stock market. Note that short-selling transactions and the use of structured products may help adjust risk exposures to the stance of the business cycle11.

2.3 The non-linear local projection method

In addition to the standard linear VAR model which is used as the benchmark in our study, we rely on a nonlinear local projection (LP) algorithm to compute the impulse response functions of hedge fund return moments—i.e., beta, co-skewness and co-kurtosis—to macroeconomic and financial shocks. Shocks related to macroeconomic and financial variables may be regarded as “external” shocks to hedge fund strategies, while shocks related to their return moments can be referred to as “internal” shocks. To get a better grasp of the LP method, we will first examine its linear version. In the linear LP method, the coefficients are not computed recursively, as in the standard linear VAR, but they are rather based on the following expression of an optimal forecast (Jordà, 2004, 2005; Misina and Tessier, 2008; Tessier, 2015) which, incidentally, corresponds to the general definition of an impulse response (Hamilton, 1994):

\[ IR(t, s, \mathbf{d}_s) = E(\mathbf{y}_{t+s} | \mathbf{v}_t = \mathbf{d}_s; \mathbf{X}_t) - E(\mathbf{y}_{t+s} | \mathbf{v}_t = \mathbf{0}; \mathbf{X}_t) \quad s = 0, 1, 2, \ldots \quad (7) \]

where \( IR \) is the impulse response at time \( t+s \); \( E(\cdot) \) denotes the best mean-squared error predictor; \( \mathbf{y} \) is the \( k \times 1 \) vector of endogenous variables; \( \mathbf{X}_t = \left( \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots \right)' \); \( \mathbf{0} \) is of dimension \( k \times 1 \); \( \mathbf{v} \) is the \( k \times 1 \) vector of reduced-form disturbances, and \( \mathbf{d}_s \) is the vector of the experimental

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11 For instance, performing short sales when markets are falling may lead to a rise in co-skewness—i.e., a decrease in tail risk. But this operation requires good forecasting skills, to say the least. Moreover, according to Agarwal et al. (2017b) and Bali et al. (2017), the use of puts during crises may reduce tail risk.
shocks. An impulse response is thus a difference between two forecasts: (i) the forecast of $y_{t+s}$ when the instantaneous structural shocks are equal to $d_i$; (ii) the forecast of $y_{t+s}$ when the structural shocks are equal to $0$.

In order to compute the IRFs associated with the LP method, we must retain the last $s+1$ observations of the sample as dependent variables of the local projection—i.e., from $y_t$ to $y_{t+s}$. We regress sequentially these vectors on the same set of lagged variables—i.e., $(y_{t-1}, y_{t-2}, \ldots, y_{t-p})$. We thus obtain the following sequence of regressions (Jordà, 2004, 2005; Kilian and Kim, 2011):

$$y_{t+h} = F_1^h y_{t-1} + F_2^h y_{t-2} + \ldots + F_p^h y_{t-p} + \mu_{t+h}, \quad h = 0, \ldots, H \quad (8)$$

The corresponding structural impulse responses are:

$$\Omega_h = \varphi_h P^{-1} = F_1^h P^{-1}, \quad h = 0, \ldots, H \quad (9)$$

where $P^{-1}$ is the Cholesky triangular matrix used to retrieve the structural shocks (Sims, 1980). Note that the impulse response coefficients involve the terms $y_{t-1}$ only. Hence, they are not computed recursively, as in the classical linear VAR model. According to Jordà (2004, 2005), they are more robust to misspecifications of the data generating process, thereby avoiding an increase in the misspecification error through the nonlinear calculation of the standard VAR technique as the forecast horizon increases.

The non-linear LP method we use is based on the following transformation of the LP model (Jordà, 2004, 2005):

$$y_{t+s} = B_1 y_{t-1} + B_2 y_{t-2} + \ldots + B_p y_{t-p} + Q_1 y_{t+s} + \xi_{t+s}, \quad s = 0, 1, 2, \ldots, h \quad (10)$$

where the observations from $t$ to $t+h$ of our time series are used to compute the IRFs. We thus introduce a quadratic term on the first lag of $y_{t-1}$ to account for the possible nonlinearities linking the endogenous variables.

Adding quadratic terms in the LP model allows accounting for the conditional volatility of the endogenous variables—i.e., for uncertainty. To observe this, assume that the time series $z_t$ is a pure random variable—i.e., $z_t = \varepsilon_t$, and that the conditional variance ($\sigma^2_t$) of $\varepsilon_t$ follows an ARCH(1) process:

$$\sigma^2_t = \alpha + \beta \varepsilon^2_{t-1} = \alpha + \beta z^2_{t-1}. \quad (11)$$

This simple example shows the close link between the conditional variance of $z_t$ and $z^2_{t-1}$.

The IRFs associated with equation (10) are easily computed as follows:

$\text{To obtain the structural impulse response, we must post-multiply equation (7) by the Cholesky triangular matrix } P^{-1}.\text{ We neglect the constant term in this equation.}$

$\text{As mentioned by Kilian and Kim (2011), Jordà (2005) does not make an explicit distinction between structural and reduced-form impulse responses.}$

$\text{Note that we can add quadratic terms on lagged values of higher order and also introduce cubic terms in equation (10). However, the formulation given by equation (10) performs the best in the context of our study. Moreover, as noted by Jordà (2004, 2005), since the impulse response coefficients involve only the terms $y_{t-1}$ in the local projection technique, it seems parsimonious to restrict nonlinearities to the first lag alone.}$
where $\text{IR}$ are the impulse response values. The impulse responses thus depend on the history of the time series as measured by $y_{t-1}$. For convenience, they can be evaluated at the mean of the time series $\bar{y}_{t-1}$. Equation (11) thus becomes:

$$\text{IR}(t,s,d_i) = \hat{B}_i^s d_i + \hat{Q}_i^s (2\bar{y}_{t-1} d_i + d_i^2)$$

As usual, the reduced-form shocks have to be orthogonalized before plotting the IRFs of the endogenous variables.

The confidence interval (CI) of the IRFs computed for non-linear LP is easily obtained. The $(1-\alpha)/2$ confidence interval of the IRF is equal to:

$$CI = IR \pm \left[ z_{1-\alpha/2} \times (w_i^c \Sigma w_i) \right]$$

where $w_i = (d_i, 2\bar{y}_{t-1} d_i + d_i^2)$, $\Sigma_c$ is the HAC variance-covariance matrix of the coefficients, and $z_{1-\alpha/2}$ denotes the $(1-\alpha/2)$-quantile of the N(0,1) distribution.

The non-linear LP model presents many advantages over the classical linear VAR procedure—i.e, asymmetry, non-proportionality and state dependence (Jordà, 2004, 2005; Misina and Tessier, 2008). In this respect, in the linear VAR model, shocks are symmetric when they change sign, in the sense that responses to positive and negative shocks are mirror images of each other (Judge et al., 1988). By contrast in the non-linear LP model, the impact of a shock depends on its sign: two shocks of equal amplitude have not necessarily the same impact in absolute value if their sign differs. Moreover, in the linear VAR model, shocks are proportional—i.e., the shocks are proportional to the change in the exogenous variables. This is obviously not the case in the nonlinear LP method, where the shocks vary with the size of the exogenous variable. Finally, the IRFs resulting from the linear VAR model are not state dependent. This is not the case for the non-linear LP model as evidenced by equation (12). None of the restrictions imposed to shocks by the linear VAR are desirable in the context of our study. Indeed, it is well-known that the behavior of economic (financial) time series is asymmetric—i.e., these series react more to negative than to positive shocks. A negative shock will also produce a different effect on hedge fund risk depending on whether the economy is in expansion or recession.

In our linear VAR model, we regress each moment of a hedge fund strategy—i.e., beta, co-skewness and co-kurtosis—on the variables of our canonical asset pricing model (equation (6)). In our nonlinear LP approach, this equation becomes:
\[ y_i = \varphi_0 + \varphi_1 \text{output}\_\text{gap} + \varphi_2 \text{output}\_\text{gap}^2 + \varphi_3 \text{credit}\_\text{spread} + \varphi_4 \text{credit}\_\text{spread}^2 + \ldots \\
+ \varphi_5 \text{term}\_\text{spread} + \varphi_6 \text{term}\_\text{spread}^2 + \varphi_7 \text{VIX} + \varphi_8 \text{VIX}^2 + \xi_i \]  

(14)

In equation (14), the non-squared variables are associated with risk and the squared or quadratic variables are associated with uncertainty (noise).

3. Data and stylized facts

3.1 Data

Data on quarterly hedge fund returns\(^{16}\) are drawn from the database managed by Greenwich Alternative Investment (GAI). GAI manages one of the oldest hedge fund databases, containing more than 13,500 records of individual hedge funds. Returns provided by the database are net of fees. Our database runs from the first quarter of 1988 to the second quarter of 2016, for a total of 116 observations—a reasonable number on which to perform our VAR analyses. In addition to the weighted composite index, our database includes 11 strategies. The description of these strategies appears in Table 1. Data for U.S. macroeconomic and financial variables are taken from the FRED database which is managed by the Federal Reserve Bank of St. Louis\(^{17}\).

3.2 Descriptive statistics

Table 2 provides the descriptive statistics of hedge fund strategies included in our database. During the entire sample period, the annual return of the hedge fund weighted index, henceforth general index, is higher than the one of the S&P500—i.e., 8.79% vs 6.68%\(^{18}\). The average returns displayed by the strategies are quite comparable. However, two strategies succeed in delivering an annual return higher than 10% over the period of analysis: growth and value index. The standard deviation of their returns is also much higher than the average. The returns of the futures and short-sellers strategies have also a standard deviation which exceeds

\[^{16}\] In this study, we rely on quarterly data for two reasons. First, the GAI quarterly database spans a longer period than the monthly database. Second, since our empirical methodology is based on VARs, relying on quarterly data leads to less noise in the computation of the impulse response functions than resorting to monthly data.

\[^{17}\] Regarding the biases related to the use of hedge fund data, see Racicot and Théoret (2016). According to Asness et al. (2001), some biases like the one due to the practice of return smoothing in the hedge fund industry (Getmansky et al., 2004) should be less serious for quarterly data—which is the frequency used in this paper—than for monthly data—i.e., the most popular frequency in hedge fund studies.

\[^{18}\] However, since 2010, the S&P 500 has shown a better performance than the hedge fund general index, the average annual returns being 9.92% and 3.84%, respectively.
the average one. Incidentally, these strategies reported substantial positive returns during the subprime crisis—i.e., 8.61% and 8.86% respectively. This performance is associated to a great extent with the short sales of these two strategies. Note however that the average annual return of the short-sellers strategy, at -2.63%, is negative over the entire sample period. Indeed, the return of short-sellers is usually negative when the stock market is trending upward.

Given the “hedging operations” of hedge funds, their beta is usually low. Indeed, the average conditional beta of the general index is equal to 0.37 over the sample period. The growth, value index and event driven strategies have the highest (positive) betas over the sample period, being 0.83, 0.58 and 0.43, respectively. Many strategies have average betas which are below the 0.20 mark—i.e., equity market neutral, fixed income, long-short credit and macro. Moreover, two strategies display a negative average beta: futures (-0.02), and especially short-sellers (-0.99).

Turning to higher moments, we note that the general index displayed less tail risk—as measured by skewness and kurtosis—than the market index over our sample period. Indeed, the skewness and kurtosis of the S&P500 are equal to -1.88 and 9.65, respectively, while the corresponding statistics for the hedge fund general index are 0.26 and 4.47. The strategies embedded with the highest (positive) skewness are: macro, growth and futures. Due to the subprime crisis, the fixed income and convertibles strategies have the lowest level of skewness and the highest level of kurtosis over the sample period. In other respects, the two strategies which are the most involved in short sales—i.e., short-sellers and futures—display low tail risk from 1988 to 2016.

In this article, as explained earlier, we do not focus on the standard skewness and kurtosis statistics to analyse the dynamic behavior of the higher moments of hedge fund strategies, but rather on co-skewness and co-kurtosis which are corrected for idiosyncratic risk. As noted previously, a higher level of co-skewness and a lower level of co-kurtosis are associated with lower tail (systematic) risk. In this instance, the hedge fund general index embeds less tail risk than the S&P500 (Table 2). Note that the level of a strategy’s co-kurtosis tends to co-move with its beta. In this respect, the growth, value index and event driven strategies have simultaneously the highest betas and co-kurtosis among the strategies listed in Table 2. Two strategies present a negative co-kurtosis: short-sellers and futures. These strategies are thus well-positioned to capture positive payoffs during crises (Jurczenko et al., 2006).

The general index displays a small positive co-skewness (0.0594) over the sample period while the S&P500 shows a small negative one (-0.0897). More importantly, co-skewness is not correlated with skewness at the strategy level. For instance, the growth strategy has a high
positive skewness (0.71) but it shows the highest negative co-skewness in absolute value (-1.39). We note the same relationship for the macro and short-sellers strategies. However, in line with their skewness, the futures and equity market neutral strategies display a positive co-skewness. Hence, the co-skewness and co-kurtosis measures do not classify strategies in the same order as skewness and kurtosis with respect to tail risk.

3.3 Stylized facts

Figure 1 provides the plots of co-kurtosis for key hedge fund strategies over our sample period. We note that, for most of these strategies, co-kurtosis tends to display the same pattern over the business cycle. However, the amplitude of co-kurtosis differs from one strategy to the next. For strategies with the highest betas—e.g., growth and value index—co-kurtosis has a greater amplitude than for strategies with low betas—e.g., equity market neutral. The co-kurtosis of two strategies—i.e., short-sellers and futures—moves in the opposite direction of the others. Moreover, similarly to their average betas, these two strategies have a negative co-kurtosis while the others have a positive one. They are thus well-positioned to capture positive payoffs in times of turmoil (Jurckzenko et al., 2006). More importantly, note that strategies’ co-kurtosis tend to return to 0 during recessions, which are shaded in Figure 1. This corresponds to less risk-taking by the strategies’ managers. Their behavior thus becomes more homogenous during crises, an obvious source of systemic risk in the hedge fund industry. Also evidenced by Figure 1, we note that the co-kurtosis of the hedge fund general index is usually higher than the market one but that is better controlled during the subprime crisis.

Co-kurtosis thus acts as a leverage effect. It increases during expansion and decreases in recession. Some strategies—i.e., growth and value index—rely more on this source of leverage in expansion but, in line with the others, they deleverage in recession. In this respect, Figure 2 plots the relationship between beta and co-kurtosis over the sample period for the strategies reported in Figure 1. We find that a strategy’s beta tends to be proportional to its co-kurtosis when this higher moment is positive. Therefore, co-kurtosis seems to behave as a leverage for the beta. During crises—i.e., in periods of rising uncertainty—this leverage tends to decrease for most strategies, and this move is associated with a decline in beta. In periods of expansion or of reduced uncertainty, leverage as measured by co-kurtosis increases, leading to a rise in beta.

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19 In order to avoid overloading the figure, we have not included all the strategies which are listed in Table 1. The amplitude of the co-kurtosis of the distressed and long-short credit strategies is close to the general index one (gi) while the co-kurtosis of the macro and fixed income strategies co-move very closely with the equity market neutral one.
This relationship also holds for the equity market neutral strategy, which is usually considered as non-directional (Agarwal et al., 2017a). Figure 2 shows that its beta and co-kurtosis are clearly procyclical (Duarte et al., 2007; Patton, 2009). However, since this strategy aims at minimizing its market exposure in the long-run, the fluctuations of its return moments are subdued.

Two strategies are quite different from the others in terms of the fluctuations of their beta and co-kurtosis: short-sellers and futures. Similarly to the other strategies, their beta and co-kurtosis tend to decrease in absolute value during recessions (Figure 2). However, during normal times, the co-movements between their beta and co-kurtosis are less tight than the ones observed for the other strategies.

Figure 3 displays the co-skewness plots for the same strategies which were selected to build the previous figures. We note much less regularity in the behavior of co-skewness from one strategy to the next than it was the case for beta and co-kurtosis. The co-skewness of the hedge fund general index is generally stable and close to 0. The co-skewness of the S&P500 is much more unstable and tends to deteriorate sometimes substantially—especially during crises. Turning to strategies, the value index and futures have quite favourable patterns for their co-skewness which remains positive most of the time. In contrast, the co-skewness of growth and short-sellers are usually negative and high in absolute value. While the co-kurtosis of the equity market neutral strategy is quite stable, its co-skewness fluctuates much more and may drop considerably. This result is in line with Duarte et al. (2007) who find, contrary to expectations, that the equity market neutral strategy may be risky during crises.

More importantly, co-skewness appears less manageable than co-kurtosis and beta during crises. In this respect, during the subprime crisis, short-sellers and growth strategies succeed in increasing their co-skewness, whereas, similarly to the market index, the co-skewness of the value index and the equity market neutral strategies have deteriorated. Hedge funds must thus make an arbitrage between the moments of their return distributions.

It may be interesting to examine how the standard measures of kurtosis and skewness—which include idiosyncratic risk—behaved during the subprime crisis in comparison with the corresponding measures of co-kurtosis and co-skewness. Figure 4 shows that kurtosis increased markedly during the subprime crisis for the general index and three selected strategies—i.e., growth, value index and equity market neutral. However, except for this last strategy, kurtosis decreased sharply after its peak. Kurtosis thus seems to be less tractable than co-kurtosis which started to decrease at the beginning of the subprime crisis. The representative hedge fund
appears unable to smooth the shock related to idiosyncratic risk embedded in kurtosis. However, kurtosis decreased for the futures strategy during the subprime crisis and was quite under control for the short-sellers strategy, in line with our previous observations on these two strategies. This fact is consistent with Asness et al. (2001) who find, over their shorter sample period, that the futures strategy display good market-timing skills\textsuperscript{20}.

The behavior of hedge fund risk associated with skewness is similar to the one related to kurtosis (Figure 5) during the subprime crisis. For the general index, the growth and value index strategies, skewness deteriorated markedly during the first part of the subprime crisis and recovered substantially thereafter: shocks were thus quickly absorbed. In line with its kurtosis, the skewness of the equity market neutral strategy continued to deteriorate after the subprime crisis, which suggests that this strategy may have difficulties to smooth its skewness following macroeconomic and financial shocks (Duarte et al., 2007; Patton, 2009). However, the skewness of the futures and short-sellers strategies evolved favourably during the crisis, suggesting once more that these two strategies may help hedging a portfolio during a crisis by endogenizing\textsuperscript{21} the impact of negative shocks.

4. Impulse response functions computed with the benchmark linear VAR and the non-linear local projection method

In this section, we study the IRFs of hedge fund risk measures—beta, co-kurtosis and co-skewness—computed with the nonlinear local projection method (LP) first over an expansion period—i.e., from 2013 to 2016— and then over the subprime crisis—i.e., from 2007 to 2010. Since the presence of squared lagged variables introduces noise in the estimation process, we select a 90% confidence interval to test the significance of the IRFs. Moreover, to analyze the impact of nonlinearities on the computation of the IRFs, we also report the IRFs built with the benchmark linear VAR procedure.

After many experiments on different orderings involving the classical information criteria to select a VAR model—i.e., the AIC, AIC\textsuperscript{22} and SIC—we order the variables of our setting in the following way:

\[ \text{output gap} \rightarrow \text{term spread} \rightarrow \text{credit spread} \rightarrow \text{risk measure} \rightarrow \text{VIX} \]

\textsuperscript{20} i.e., the managed futures strategy loads positively on the squared market return.
\textsuperscript{21} These strategies may endogenize external shocks by an opportune use of derivatives or by synchronized short sales.
\textsuperscript{22} The AIC\textsubscript{c} is a corrected version of the AIC criterion proposed by Hurvich and Tsay (1993) and specifically designed for VARs. See also Jordà (2005).
The output gap is thus considered as the most exogenous variable while the VIX, which is very sensitive to economic news, is viewed as the most endogenous. At the impact, the risk measure reacts to the output gap, the term spread and the credit spread but not to the VIX. Note that this ordering is empirical since there does not exist a comprehensive theoretical framework on the co-movements between the variables in our VAR model.

### 4.1 IRFs over an expansion period (2013-2016)

#### 4.1.1 IRFs of strategies' betas

For our simulations over the expansion period, on the basis of the usual information criteria—i.e., the AIC, AICc and SIC criteria—we compute the impulse response functions of our models with three lagged values for our vector of explanatory variables. Figure 6 provides the results of our experiments for the betas of the hedge fund general index and for the selected strategies. We note that the most significant IRF for the general index beta is obtained with the output gap. The beta is quite procyclical and this relationship does not seem to depend on nonlinearities since the IRF obtained with the standard VAR is very similar to the one generated by the nonlinear LP method. As expected, the representative hedge fund thus takes more systematic risk in expansion—i.e., its beta increases. According to Figure 6, the beta of most strategies is also procyclical—i.e., it reacts positively to an output gap shock. This procyclicality is particularly strong for the growth, macro, event driven and value index strategies. Surprisingly, the beta of the equity market neutral strategy is also quite procyclical even if this strategy aims at hedging market risk in the long-run (Duarte et al., 2007; Patton, 2009).

However, two kinds of strategies differ: those more involved in fixed income securities—i.e., fixed income and convertibles—and those which focus more on short sales—i.e., futures and short-sellers. The first category seems to have a countercyclical beta, probably because the increase in interest rate which is usually associated with a rise in the output gap is detrimental to the business lines of these strategies. In other respects, the beta of the futures strategy takes a long time before increasing following an output gap shock and this increase is much less significant for this strategy than for the representative hedge fund. For short-sellers, we note that its beta seems countercyclical: it decreases following an output gap shock. But being negative, it increases in absolute value, which also corresponds with more systematic risk for short-sellers.

The beta associated with the general index decreases after a VIX shock, a term spread shock and a credit spread shock. The representative hedge fund appears to manage the risk
associated with these shocks. Once more, the strategies which are the most procyclical show the same reaction—i.e., value index, growth, macro and event driven. However, for some strategies, the response of their beta to these shocks may be quite different. Actually, strategies quite involved in short sales seem to increase their beta after a VIX shock. In this respect, the beta of the futures strategy tends to increase and the beta of short-sellers decreases very significantly—i.e., increases in absolute value. Indeed, according to the Black (1976) leverage effect, an increase in the VIX is associated with falling stock markets, which benefits short-sellers. Not surprisingly, a rising VIX has a weak impact on the betas of the equity market neutral strategies and of strategies focusing on bond markets—especially fixed income and convertibles. Moreover, a positive term spread shock induces strategies which focus more on bond markets—i.e., convertibles, distressed, fixed income and long-short credit—to increase their beta. They try to capture the liquidity premium associated with the term spread in times of expansion, while economic uncertainty is low (Lambert and Plantania, 2016). Similarly, the beta of three strategies increases after a credit spread shock: distressed, fixed income and futures. In times of low uncertainty, these strategies which embed a great deal of credit risk are induced to capture the credit spread. This behavior is consistent with Agarwal et al. (2017b) who argue that the payoffs of many hedge fund strategies behave as short puts in good times. For instance, strategies may capture the credit spread by selling credit default swaps (CDS). Thus, they act as insurance sellers in economic expansion. According to Bali et al. (2017), the kurtosis of a short put computed in the risk-neutral universe decreases in good times while its skewness also decreases. There is thus a trade-off between the higher moments of the strategies which behave as short puts in good times.

Summarizing, in expansion, hedge funds tend to adjust their beta to shocks either to manage their risk or to increase their payoffs depending on the nature of their business lines. During good times, the output gap shock has the most important impact, the representative hedge fund increases its beta to capture the gains associated with increased economic growth. Although the beta of the representative hedge fund tends to decrease following a VIX shock, a term spread shock and a credit spread shock, some specialized strategies adopt an opposite behavior to benefit from these shocks. For instance, an increase in the VIX is profitable for strategies involved in short sales—e.g., futures and short-sellers. Strategies focusing more on bond markets—i.e., convertibles, distressed, fixed income, and long-short credit—tend to increase their beta in expansion in order to capture the premia associated with term spread and credit spread shocks. Indeed, the risk associated with this move is lower in expansion.

[Insert Figure 7 here]
4.1.2 IRFs of higher moments

4.1.2.1 Co-kurtosis

Since a strategy beta tends to co-move tightly with its co-kurtosis, Figure 7 provides the IRFs of the general index and of some key strategies only. Not surprisingly, the pattern of the IRFs of the general index co-kurtosis is very similar to its beta, albeit less significant since the behavior of co-kurtosis is less smooth than the beta. Co-kurtosis is procyclical—i.e., the representative hedge fund takes more fat-tail risk in expansion. In line with the beta, the hedge fund general index co-kurtosis responds negatively to VIX and credit spread shocks. Turning to strategies, we note that co-kurtosis responds more to shocks for strategies with a higher beta—e.g., growth—than for strategies with a lower beta—e.g., equity market neutral and futures. Therefore, as discussed previously, strategies with a higher beta bear more fat-tail risk. Interestingly, even if futures and short-sellers have both negative co-kurtosis, they do not respond in the same direction to an output gap shock. While the co-kurtosis of the futures strategy tends to increase smoothly following such a shock, the co-kurtosis of short-sellers declines sharply and very significantly. In absolute value, the co-kurtosis of the futures strategy is thus countercyclical while the one of short-sellers is procyclical. It seems that the futures strategy may provide better diversification benefits in recession since the more negative the co-kurtosis of a fund is when stock markets fall, the more well-positioned it is to capture positive payoffs (Jurckzenko et al., 2006).

4.1.2.2 Co-skewness

It is well-known that hedge funds must trade-off higher moments when constructing optimal portfolios. They cannot simultaneously increase their return and co-skewness while decreasing their beta and co-kurtosis. The standard efficient frontier obtained when trading higher moments is thus lower than the one computed with only the first two moments of the return distribution. In our setting, this necessary arbitrage seems to be performed at the expense of risk associated with co-skewness. In Figure 8, we note that the co-skewness of the general index improves markedly after a positive output gap shock, which corresponds to a decrease in the level of tail risk. However, it deteriorates after VIX and credit spread shocks. Therefore, there is an arbitrage or trade-off between beta and co-kurtosis, on the one hand, and co-skewness, on the other hand. Following these shocks, the representative hedge fund succeeds

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\[\text{I.e., hedge funds must trade-off return against lower tail risk.}\]
in reducing its risk associated with its beta and co-kurtosis but this reduction in risk is at the expense of increased risk related to co-skewness.

The co-skewness of the value index responds in a similar way to shocks as the one of the general index. However, the growth strategy behaves quite differently. Its co-skewness responds negatively to an output gap shock, suggesting that it is countercyclical. Also, in contrast to the general index, it responds positively to VIX and credit spread shocks. This pattern is consistent with the findings of Campbell et al. (2010) who argue that growth stocks display hedging properties with regards to market reversals. The IRFs of the macro strategy suggests that it may have the same properties.

Similarly to growth and macro, the co-skewness of strategies for which it is countercyclical tends to respond positively to adverse VIX and credit spread shocks, which is associated with a decrease in tail risk. It is especially the case for strategies focusing on the bond market: distressed, event-driven and fixed income. This result is not surprising since the performance of the bond market is countercyclical. The co-skewness of the futures and short-sellers strategies, which share many similarities, responds negatively to a VIX shock. However, the co-skewness of short-sellers responds negatively to an output gap shock while the futures strategy reacts positively. Finally, the co-skewness of strategies which benefit from a term spread shock—especially distressed, fixed income and long-short-credit—decreases following such a shock, suggesting that capturing the term-spread does not go without some risk. Once more, this result is in line with Agarwal et al. (2017b) who assert that many hedge fund strategies behave as insurance sellers during good times by deliberately increasing their tail risk.

4.2 IRFs over the subprime crisis (2007-2010)

To better grasp the effects of nonlinearities on the behavior of hedge fund multimoment risk, we also perform a nonlinear local projection of our model over the subprime crisis, which stretches from the second quarter of 2007 to the fourth quarter of 2010. In line with Bekaert and Hodrick (2012), we also consider the year 2010 as part of the subprime crisis even if the U.S. recession officially ended in 2009 according to the National Bureau of Economic Research (NBER). Indeed, the subprime crisis had protracted effects and the year 2010 was not satisfying the norms of a strong recovery. In this respect, the information criteria indicate that the optimal lag of the endogenous variables increases during the subprime crisis compared to the expansion period. On the basis of these criteria, we select a lag equal to six quarters for the crisis period, which suggests that the impact of shocks lasts longer during crises than during
normal times. We also use a lag of six quarters for the explanatory variables of our benchmark linear VAR model.

[Insert Figure 9 here]

4.2.1 IRFs of strategies’ betas

During the subprime crisis, the IRFs associated with the beta of the general index evolve in the same direction as in expansion following macroeconomic and financial shocks (Figure 9). However, the responses of the beta are quicker, more important, and last longer. The confidence intervals of the IRFs are also tighter. Hence beta definitively decreases after a negative output gap shock. The impact of this shock is more stretched over this projection period than over an expansion and displays cycles when using the LP method. Moreover, VIX and credit spread shocks lead to a drastic reduction in the beta of the representative hedge fund in the short-run. This response is almost exclusively driven by nonlinearities since the magnitude of the IRFs associated with the benchmark VAR is very low compared to the nonlinear LP method. But according to the forecast of the beta provided by the LP method, this measure of risk reverses its downward movement after four quarters, suggesting that hedge funds may have difficulties in timing their systematic risk during a crisis.

Turning to strategies, we note that after a negative output gap shock they usually decrease their beta during the projection performed over the period associated with the subprime crisis, and that the corresponding IRFs are tighter over this period. In other respects, the difficulty to track the impact of VIX and credit spread shocks on the beta is particularly high for some strategies which are confronted to a quick and significant upward reversal of their beta after these shocks—i.e., event driven, equity market neutral, futures, value index, and growth. In expansion, risk management was easier to implement. Interestingly, the effect of the term spread shock is usually higher and more significant in the subprime crisis than during the expansion period. This result may be related to the liquidity constraint which is more at play during crises. For instance, the beta of some strategies—i.e., distressed, event driven, equity market neutral, fixed income, growth, short-sellers, and value index—jump after a term spread shock, which suggests that the deterioration of their liquidity position increases the systematic risk they bear. However, portfolio managers of a majority of these strategies seem to regain the control of their beta after six quarters. Interestingly, the macro strategy succeeds quite well in mastering risk related to a term spread (liquidity) shock.

4.2.2 IRFs of higher moments

4.2.2.1 Co-kurtosis
Similarly to the expansion period, we reproduce only the IRFs of the co-kurtosis of some key strategies since the co-kurtosis of a strategy usually co-moves tightly with its beta. In line with the beta, a negative output gap\textsuperscript{24} shock leads to a decrease in the co-kurtosis of the general index during the subprime crisis. The VIX and credit spread shocks have a much more important and significant negative effect on the general index co-kurtosis in the crisis than in expansion, and similarly to the beta, this effect is mainly driven by nonlinearities. Once more, we note that hedge funds may have difficulties in keeping their control on co-kurtosis, while tracking risk is easier during expansion.

While the response of the general index beta to a term spread shock was mitigate during the projection performed over the subprime crisis, its co-kurtosis responds strongly to it in the short-run. A liquidity shock as measured by the term spread thus entails a rise in hedge fund co-kurtosis risk at the impact. However, this effect is reversed after four quarters whereas co-kurtosis decreases very significantly, suggesting that hedge funds regain the control over their fat-tail risk after a liquidity shock.

Regarding strategies, the same comments apply to the plots of their co-kurtosis than to the ones of their betas. Strategies having a positive co-kurtosis in Figure 10—i.e., equity market neutral, growth and macro—reduce significantly their fat-tail risk following a negative output gap shock\textsuperscript{25}. In the short-run, VIX and credit shocks give way to a decrease in the co-kurtosis of these strategies which is important and more significant than during the expansion period. However, co-kurtosis resumes its upward trend after four quarters. Similarly to the general index, the co-kurtosis of these strategies jumps temporarily after a term spread shock. Thus, they have difficulties in controlling liquidity risk in the short-run during a crisis. Finally, short-sellers and futures strategies display co-kurtosis IRFs plots which are the opposite of the ones of the general index and of the strategies appearing in Figure 10.

4.2.2.2 C-skewness

Turning to the behavior of co-skewness during the subprime crisis, we find many similarities with the plots of the IRFs obtained in expansion (Figure 11). Indeed, they generally evolve in the same direction in expansion as during the subprime crisis. However, in line with co-kurtosis, the responses of co-skewness to shocks are stronger, quicker and more significant,

\textsuperscript{24} In contrast to the expansion period, we consider only negative output gap shocks during the subprime crisis.

\textsuperscript{25} Similarly to the expansion period, the magnitude of this reduction is proportional to the level of the beta of the corresponding strategy.
the confidence intervals of the IRFs computed during the crisis being generally much tighter than in expansion.

A negative output gap shock has a detrimental effect on the general index co-skewness and the responses computed with the benchmark VAR and the nonlinear LP are comparable, although the response associated with the LP method is cyclical and is more protracted. Even if hedge funds succeed in reducing their beta and co-kurtosis during the crisis—at least temporarily—they must arbitrage these moves with co-skewness risk.

Subsequently to VIX and credit shocks, the co-skewness of the general index decreases, which corresponds to a rise in the level of tail risk. Nonlinearities clearly dominate these responses. However, after four quarters, hedge funds regain the control over their co-skewness since it becomes significantly positive. Interestingly, while a term spread shock does not impact significantly co-skewness in expansion, this kind of shock influences positively and significantly the general index co-skewness in the short-run. A similar reaction is shared by the value index, futures and short-sellers strategies. Exploiting the mispricing of stocks or focusing on short sales may thus benefit from a reduction in market liquidity.

Strategies also react more strongly to shocks during the local projection performed over the subprime crisis than during the expansion period. Similarly to the expansion period, most strategies whose co-skewness is countercyclical—i.e., distressed, event-driven, equity market neutral, growth and macro—present very interesting dimensions for a risk-averse investor during a crisis. Following a negative output gap shock, the co-skewness of these strategies increases, suggesting that they can act as good hedges during recessions or crises. Moreover, in the short-run, their co-skewness reacts positively to VIX and credit spread shocks, which makes them even more attractive for risk-averse investors. This favourable behavior of the co-skewness of these strategies may be explained by the important weight of bond transactions in their business lines—e.g., distressed, event-driven—by the relative skills of their managers—i.e., macro—or by the hedging power of their operations—i.e., equity market neutral and growth. Note that the fixed income strategy also displays a countercyclical co-skewness but that it was particularly affected by the subprime crisis, being too invested in mortgage-backed securities. Therefore, our forecast shows a deterioration of its co-skewness after a VIX shock and especially after a credit shock.

In our experiments performed over an expansion period, the term spread has poor explanatory power in the co-skewness equations of the strategies. Indeed, in expansion, the liquidity constraint is usually not binding. It is not the case during the subprime crisis. In the short-run, a term spread shock leads to a significant decrease in the co-skewness of strategies which were insensitive to this kind of shock in expansion—i.e., event driven, equity market
neutral, growth and macro. A decrease in the liquidity of the financial markets is thus detrimental to these strategies from the point of view of co-skewness risk.

4.2.3 Discussion

Let us summarize the main results obtained from our experiments during the subprime crisis. First, the impact of macroeconomic and financial shocks on hedge fund multi-moment risk is much longer during the crisis than during the expansion period. Second, nonlinearities are much more at play during the crisis—especially for VIX and credit spread shocks—regardless of the risk measure. Third, the IRFs associated with shocks are generally much more significant and important during the subprime crisis. Fourth, monitoring multi-moment risk seems more difficult during the crisis. In many occasions, hedge fund may lose temporarily the control over their measures of risk. Overall, during crises or recessions, hedge funds seek to reduce their tail risk by becoming insurance buyers or “hedgers” but this alchemy requires skill, to say the least (Agarwal et al., 2017b). Our results show that some strategies succeed better in transforming the profile of their payoffs from short to long puts when transiting from expansion to crisis, or, in other terms, to transform their business lines from insurance sellers in order to capture risk premia to insurance buyers in order to hedge their operations. According to Bali et al. (2017), the kurtosis of puts decreases in bad times while their skewness also decreases. During crises, this behavior is desirable because investors mostly fear a large negative payout if an extreme event occurs. The decrease in the kurtosis of puts on the S&P500 was particularly important during the subprime crisis and especially during the Lehman Brothers bankruptcy in September 2008, an obvious extreme event.

5. Systemic risk related to the moments of the hedge fund strategies’ return distributions

If financial institutions behave more homogenously as regards to risk when macroeconomic and financial uncertainty increases, systemic risk—i.e., risk related to contagion among financial institutions—rises in the financial system. For instance, during crises, financial institutions may deleverage simultaneously, which leads to fire sales of assets (Shleifer and Vishny, 2010; Gennaioli et al., 2013). This contributes to decrease the net worth of these institutions and increase the probability of bankruptcies, with detrimental spillover effects on the economy.

Using a methodology popularized by Beaudry et al. (2001) and Baum et al. (2002, 2004, 2009), Racicot and Téoret (2016) find that the cross-sectional dispersion of the betas of hedge
fund strategies tends to decrease in times of rising macroeconomic and financial uncertainty—an obvious source of systemic risk in the hedge fund industry. In this study, we go a step further and analyze the factors which may explain the more homogenous (or heterogeneous) behavior not only of strategies’ betas, but also of their higher moments—i.e., co-skewness and co-kurtosis. We still rely on equation (14) to implement our experiments. On the LHS of this equation, \( y_t \) becomes the cross-sectional dispersions of the strategies’ betas, co-skewness and co-kurtosis. These dispersions are simply the standard deviations of these measures of risk computed using our eleven strategies over each quarter of our sample. As previously, we rely on our benchmark VAR model and nonlinear LP method to run our regressions.

Figure 11 provides the plots of the cross-sectional dispersions of our strategies’ risk measures from the first quarter of 1988 to the second quarter of 2016. Similarly to Racicot and Théoret (2016), the cross-sectional dispersion of strategies’ betas tends to decrease sharply during a recession and they even anticipated the tech-bubble and subprime crises. More importantly, the cross-sectional dispersion of strategies’ co-skewness, and especially strategies’ co-kurtosis, display the same pattern during crises. Interestingly, the cross-sectional dispersion of co-kurtosis tended to zero during the tech-bubble and subprime crises. It appears that hedge fund strategies become more homogenous with respect to risk during recessions or crises, regardless of the risk measure used.

Figure 12 transposes our VAR methodology to the strategies cross-sectional dispersion of our three risk measures. We note that the output gap shock is quite important for explaining the dynamics of the beta dispersion. In line with Figure 11, this shock contributes to increase the beta dispersion in expansion but to reduce it during the subprime crisis. A VIX shock has a stronger impact on the cross-sectional dispersion of beta in crises and we note that nonlinearities are more at play during turmoil. While this shock tends to increase the cross-sectional dispersion of the beta in expansion, it decreases it temporarily in crises. However, the dispersion rebounds thereafter. A VIX shock thus makes the behavior of hedge funds more homogenous in terms of the beta at the start of a crisis but this move reverses quickly. Albeit less important, a credit spread shock has a similar effect on the beta dispersion in expansion and during crises. Finally, a term spread shock makes strategies more heterogeneous in terms of the beta, both in expansion and during crises.

Turning to co-kurtosis, we note that the output gap is the only shock having a substantial impact on its cross-sectional dispersion in expansion. Similarly to the beta, this dispersion is procyclical. However, during the subprime crisis, the response of the co-kurtosis cross-sectional dispersion to shocks is much stronger and significant than for the beta.
dispersion. A VIX shock has a longer negative impact on the co-kurtosis dispersion than on the beta dispersion, and nonlinearities dominate the plot. Moreover, this dispersion responds negatively to a credit spread shock and, once more, this reaction lasts longer than in the case of the beta dispersion. Therefore, in the subprime crisis, shocks tend to make the behavior of strategies more homogenous in terms of co-kurtosis, which thus represents a major source of systemic risk. And even if a term spread shock increases the co-kurtosis cross-sectional dispersion at the impact, this behavior reverses quickly and significantly. Finally, the cross-sectional dispersion of strategies co-skewness displays similar responses to shocks than co-kurtosis, both in expansion and crises.

Summarizing, the cross-sectional dispersions of our three risk measures are clearly procyclical—i.e., they respond positively to an output gap shock. This macroeconomic shock thus contributes to make the behavior of strategies more heterogeneous in expansion but more homogenous in recessions or crises. Apart from the output gap shock, a VIX shock is another important factor driving the cross-sectional dispersions. While this kind of shock tends to increase the cross-sectional dispersion of beta in crises, it tends to decrease the dispersions of the two other measures of risk, at least in the short-run. We note the same pattern when considering a credit spread shock. Therefore, tail risk seems to be a major source of systemic risk in the hedge fund industry. Focusing only on the behavior of the beta cross-sectional dispersion may lead to a severe understatement of systemic risk in this industry during crises.

6. Robustness check

In order to better assess the asymmetries and nonlinearities related to the responses of hedge fund measures of risk to macroeconomic and financial shocks, we also compute the IRFs using the methodology of threshold VAR developed by Balke (2000). The structural threshold VAR (TVAR) reads as follows:

$$Y_t = A^1 Y_t + B^1 (L) Y_{t-1} + \left[ A^2 Y_t + B^2 (L) Y_{t-1} \left( I(c_{t,d} > \gamma) \right) \right] + U_t$$

(15)

where $Y_t$ is a vector containing the variables which constitute the VAR; $B^1(L)$ and $B^2(L)$ are lag polynomial matrices in the two regimes, and $U_t$ are structural innovations. $c_{t,d}$ is the threshold variable which determines in which regime the system is and $I(c_{t,d} > \gamma)$ is an indicator function which takes the value of 1 if $c_{t,d} > \gamma$ and 0 otherwise. $A^1$ and $A^2$ are associated with the structural contemporaneous relationships in the two regimes, respectively.

Since the TVAR given by equation (15) is nonlinear, the amplitude of the corresponding IRFs depends on the sign of a shock and varies nonlinearly with the size of this shock. In this respect, Balke (2000) computes the IRFs corresponding to positive and negative shocks for the
two regimes which he defines—i.e., a low and a high regime. For each regime, he also defines IRFs corresponding to two shocks of different size: (i) a moderate (standard shock) corresponding to one standard deviation of the corresponding innovation (+1SD); (ii) a big shock, defined as two standard deviations of the innovation (+2SD).

As argued by Balke (2000), if the threshold value $\gamma$ were known, the test of the significance of the threshold would be $A^2 = B^2(L) = 0$ under the null hypothesis. However, $\gamma$ is not known. Therefore, the threshold model is estimated by ordinary least squares for all possible $\gamma$ values. For each $\gamma$, the Wald statistic of no difference between regimes is computed. To test the behavior of the threshold value, three statistics are computed: (i) sup-Wald, the maximum Wald value obtained over all simulations; (ii) avg-Wald, the average Wald value over all possible threshold values; (iii) exp-Wald, a function of the sum of exponential Wald statistics. As suggested by Koop et al. (1996) and Pesaran and Shin (1997, 1998), we rely on the generalized impulse response function to identify the structural shocks in equation (15) since it is nonlinear. Finally, we use three lags for each variable to estimate the TVAR.

Insert Table 3 here

Figure 14 displays the IRFs associated with TVAR for our three measures of risk computed using the hedge fund general index while Table 3 provides the corresponding Wald tests. Being close to 0, the threshold value associated with beta separates quite well the low and high regimes (Table 3). The Wald statistics also indicate that this value is very significant. In the low regime, after a moderate negative GDP growth shock (+1SD), the beta of the general index decreases progressively. However, at the impact of a strong shock (+2SD), the beta decreases but it increases substantially thereafter. Consistent with our previous results, hedge funds may have difficulties in managing strong negative shocks. This is an obvious asymmetry in their behavior after moderate and strong GDP shocks. In other respects, again in the low regime, we note an asymmetry between moderate and strong positive GDP shocks in the sense that the beta responds more than proportionally to a strong shock. This asymmetry suggests that the financial constraint is binding in the low regime and that it has an important impact on the risk-taking behavior of hedge funds. A strong positive shock to GDP loosens the financial constraint of hedge funds, so they can take more risk—i.e., increase their beta. It is much less the case for a moderate shock.

Insert Figure 14 here

In the high regime, we observe less asymmetries in the IRF plots of the beta. However, the beta decreases more after a negative GDP shock than it increases after a positive shock. The difference between moderate and strong shocks is much less important in a high regime. It is
also easier to monitor the beta after a negative GDP shock in the high regime than in the low
one.

Not surprisingly, given its close link with the beta, the co-kurtosis of the general index
is procyclical—i.e., it responds positively to a positive GDP shock and negatively to a negative
one in both regimes. However, there is a delay between the occurrence of the shock and the
response of co-kurtosis. Hence, hedge funds seem to monitor their fat-tail risk since co-kurtosis
ultimately decreases following a negative GDP shock. However, this result must be qualified. In
both regimes, hedge funds increase more their fat-tail risk after a positive GDP shock than they
decrease it following a negative shock, which suggests difficulties in managing risk when shocks
to GDP are negative. Regarding asymmetries, we note that they are less pronounced than in the
case of the beta. However, the impact of a negative GDP shock is greater in the low than in the
high regime. Thus, managing fat-tail risk seems to be more imperative in the low regime.

Turning to co-skewness, in the low regime, it is relatively stable following a negative
GDP shock during the first four quarters after the shock and it increases slightly thereafter.
The response to a big shock is close to the one corresponding to a moderate shock. This shock
thus seems to be under control in the low regime. As expected, co-skewness responds positively
to a positive GDP shock but the deterioration which follows more than offsets the initial
improvement. This pattern is much more pronounced for a big shock. In the high regime, the
IRFs display the same patterns except that the cycles have a greater amplitude. However, before
recovering, co-skewness deteriorates slightly at the impact after a negative shock.

Figure 15 repeats the same exercise reported in Figure 14 for a VIX shock. In the low
regime, the beta of the general index responds negatively to a positive VIX shock. This
suggests that hedge funds seem to manage their volatility risk. Conversely, their beta responds
positively to a negative VIX shock—i.e., hedge funds take more systematic risk when the
volatility of financial markets decreases. In the high regime, we observe the same behavior
except that the amplitude of the IRFs is less. Moreover, co-kurtosis responds in the same way as
the beta following a VIX shock. There are no obvious differences between the low and high
regime.

Regarding co-skewness, Figure 15 shows that it decreases following a positive VIX
shock and increases after a negative VIX shock. Consistent with our previous results, there is
less evidence of co-skewness monitoring during a crisis. On the asymmetry side, co-skewness
increases more than proportionally following a big negative shock than following a moderate
shock. We obtain the same results in the high regime.

26 Once more, the threshold variable is GDP growth.
7. Conclusion

In their seminal papers Mitchel and Pulvino (2001) and Agarwal and Naik (2004) have established that many hedge fund strategies—and especially those involved in arbitrage activities less captured by the CAPM model, which may be considered as non-directional—display payoffs which are similar to those of a short put. This study is in line with others which also relate the payoffs of hedge fund strategies to those of derivatives or structured products (Mitchell and Pulvino, 2001; Stafylas et al., 2017). For instance, the payoffs of the trend followers and market timers—particularly managed futures or Commodities Trading Advisors (CTAs)—may be associated with the payoffs of straight straddles or lookback straddles (Fung and Hsieh, 1997, 2001, 2002, 2004; Stafylas et al., 2017). However, these early analyses were quite static since they did not envision a change in the behavior of hedge funds according to the stance of the business cycle. Following the onset of the subprime crisis, many papers have been devoted to the asymmetric behavior of hedge funds dependent on the phase of the business cycle (Jawadi and Khanniche, 2012; Bali et al., 2014; Calmès and Théoret, 2014; Namvar et al., 2016; Racicot and Théoret, 2016; Stafylas et al., 2017; Agarwal et al., 2017b). In this respect, our study complements the work of Agarwal et al. (2017b) which is devoted to a panel analysis of the hedge fund tail risk and concerned with the micro-dynamics dimensions of the timing of tail risk in the hedge fund industry. Compared to this study, our paper relies on a holistic (time series) approach in order to analyze how macroeconomic and financial shocks impact three measures of hedge fund risk—i.e., beta, co-skewness and co-kurtosis. To the best of our knowledge, our study is the first that really focuses on the macroeconomic dimensions of tail risk in the hedge industry using a nonlinear local projection approach.

Our study, which is performed on a longer horizon than the one of Agarwal et al. (2017b), supports one of the main results of their work—i.e., the managers of hedge fund strategies tend to behave as insurance sellers in economic expansions but become hedgers or insurance buyers in recessions or crises. For instance, many strategies tend to increase their exposure to term spread and credit risk in expansion but reduce it in recession. Hence, regarding credit risk, they behave as if they were sellers of credit default swaps (CDS) in expansion and buyers of CDS in recession. However, our projection over the subprime crisis...

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27 According to this view, hedge funds “generate small positive returns most of the times before incurring a substantial loss” (Agarwal et al., 2017b). However, this pattern may be considered as somewhat “reductivist” in the sense that hedge funds are often associated with positive absolute returns and that their payoffs are usually procyclical (Racicot and Théoret, 2016). Moreover, hedge funds strategies with payoffs similar to straddles—like CTA, managed futures or macro—can produce substantial positive payoffs when stock markets are volatile—especially during crises. Short-sellers also benefit from falling stock markets.

28 Note that the payoffs of a market timer who is not involved in short sales are similar to the ones of a long call. Being also involved in short sales transforms these payoffs in straddles or lookback straddles (Stafylas et al., 2017).

29 Even if they considered that the payoffs of hedge funds were option-like or nonlinear —i.e., dynamic.
reveals that some strategies may have difficulties to control their tail risk during a crisis—especially risk associated with co-skewness. In this respect, the equity market neutral strategy—which seeks to neutralize all systematic risk by derivative securities or short sales in the long run—may be risky during crises and is thus not so neutral after all. Minimizing risk exposure by means of hedging does not always lead to the desired results (Duarte et al., 2007; Patton, 2009). This finding is also in line with Agarwal and Naik (2004) who argue that the mean-variance approach may underestimate portfolio losses and that this underestimation may be substantial for portfolios with low volatility similar to the ones held by the equity market neutral strategy. In this respect, even if hedge funds may be considered as variance reducers as argued by George Hall representing the Managed Fund Association before the U.S. House Financial Services Committee in 2007, their difficulties to control tail risk in periods of crises may offset the lower variance of their returns for investors concerned with the impact of adverse macroeconomic and financial shocks on the level of their downside risk.

Finally, we study the systemic risk in the hedge fund industry using an underpinning proposed by Beaudry et al. (2001). We find that the cross-sectional dispersions of our three hedge fund risk measures tend to decrease during crises, which suggests that the behavior of the managers of hedge fund strategies become more homogenous in times of turmoil. Thus, they tend to deleverage as a group by relying on fire sales of assets, among others, an obvious source of systemic risk (Shleifer and Vishny, 2010; Gennaioli et al., 2013). The VIX seems to be the most important factor contributing to systemic risk in the hedge fund industry. Importantly, the cross-sectional dispersions of our three risk measures seem procyclical. An increase in economic growth thus contributes to reduce systemic risk but, conversely, a decrease tends to rise it. Focusing only on the beta to track hedge fund systemic risk over the business cycle may thus lead to a serious underestimation of this kind of risk and thus on the potential of failures in the hedge fund industry. Investors and regulators should thus improve their framework for analyzing hedge fund risk, as proposed in this paper.
References


House Committee on Financial Services, Hedge Funds and Systemic risk in the Financial Markets: Hearing before the Committee on Financial Services, 100th Cong., 1st sess., 13 March 2007.


Studies 22, 2495-2530.


### Table 1 Description of hedge fund strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Convertible (CV)</strong></td>
<td>They take a long position in convertibles and short simultaneously the stock of companies having issued these convertibles in order to hedge a portion of the equity risk.</td>
</tr>
<tr>
<td><strong>Distressed securities (DS)</strong></td>
<td>The managers buy equity and debt at deep discounts issued by firms facing bankruptcy.</td>
</tr>
<tr>
<td><strong>Diversified event driven (ED)</strong></td>
<td>The manager tries to benefit from mispricings related to corporate events (e.g., spin-offs, mergers and acquisitions, bankruptcy reorganizations and share buybacks).</td>
</tr>
<tr>
<td><strong>Equity market neutral (EMN)</strong></td>
<td>The managers aim at obtaining returns with low or no correlation with equity and bond markets. They exploit the pricing inefficiencies between related equity securities.</td>
</tr>
<tr>
<td><strong>Fixed income (FI)</strong></td>
<td>The managers follow a variety of fixed income strategies like exploiting relative mispricing between related sets of fixed income securities. They invest in MBS, CDO, CLO and other structured products.</td>
</tr>
<tr>
<td><strong>Futures (FUT)</strong></td>
<td>The manager utilizes futures contracts to implement directional positions in global equity, interest rate, currency and commodity markets. He resorts to leveraged positions to increase his return.</td>
</tr>
<tr>
<td><strong>Growth</strong></td>
<td>The managers invest in companies experiencing strong growth in earnings per share.</td>
</tr>
<tr>
<td><strong>Long-short credit (LSC)</strong></td>
<td>They take long and short positions in credit in spite of the unavailability of bonds. They invest in high-yield bonds, CDS and CDO, among others.</td>
</tr>
<tr>
<td><strong>Macro</strong></td>
<td>These funds have a particular interest in macroeconomic variables. They take positions according to their forecasts of these variables. Managers rely on quantitative models to implement their strategies.</td>
</tr>
<tr>
<td><strong>Short sellers (SS)</strong></td>
<td>Managers take advantage of declining stocks. Short-selling consists in selling a borrowed stock in the hope of buying it at a lower price in the short-run. Managers' positions may be highly leveraged.</td>
</tr>
<tr>
<td><strong>Value index (VI)</strong></td>
<td>Managers invest in securities which are perceived undervalued with respect to their &quot;fundamentals&quot;.</td>
</tr>
</tbody>
</table>


### Table 2 Descriptive Statistics, Greenwich Alternative Investment hedge fund strategy returns and moments, 1988Q1–2016Q2
<table>
<thead>
<tr>
<th></th>
<th>CV</th>
<th>DS</th>
<th>ED</th>
<th>EMN</th>
<th>RI</th>
<th>FLT</th>
<th>GROWTH</th>
<th>LSC</th>
<th>MACRO</th>
<th>SS</th>
<th>VI</th>
<th>Mean</th>
<th>GI</th>
<th>MKT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return</td>
<td>0.0211</td>
<td>0.0238</td>
<td>0.0249</td>
<td>0.0351</td>
<td>0.0216</td>
<td>0.0223</td>
<td>0.0272</td>
<td>0.0177</td>
<td>0.0133</td>
<td>-0.0066</td>
<td>0.0070</td>
<td>0.0192</td>
<td>0.0220</td>
<td>0.0367</td>
</tr>
<tr>
<td>Annual</td>
<td>8.44%</td>
<td>9.54%</td>
<td>9.96%</td>
<td>7.66%</td>
<td>8.66%</td>
<td>8.92%</td>
<td>10.87%</td>
<td>7.10%</td>
<td>5.33%</td>
<td>-2.63%</td>
<td>10.80%</td>
<td>7.69%</td>
<td>8.79%</td>
<td>6.68%</td>
</tr>
<tr>
<td>Median</td>
<td>0.0241</td>
<td>0.0334</td>
<td>0.0291</td>
<td>0.0164</td>
<td>0.0230</td>
<td>0.0087</td>
<td>0.0277</td>
<td>0.0210</td>
<td>0.0106</td>
<td>-0.0142</td>
<td>0.0275</td>
<td>0.0189</td>
<td>0.0230</td>
<td>0.0228</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1500</td>
<td>0.0946</td>
<td>0.1193</td>
<td>0.1111</td>
<td>0.0815</td>
<td>0.1583</td>
<td>0.3698</td>
<td>0.0813</td>
<td>0.1931</td>
<td>return</td>
<td>0.1918</td>
<td>0.1554</td>
<td>0.1706</td>
<td>0.1205</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.2079</td>
<td>-0.1491</td>
<td>-0.1039</td>
<td>-0.0579</td>
<td>-0.0857</td>
<td>-0.1394</td>
<td>-0.1372</td>
<td>-0.0795</td>
<td>-0.1076</td>
<td>-0.2984</td>
<td>-0.1128</td>
<td>-0.1312</td>
<td>-0.0908</td>
<td>-0.3169</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0449</td>
<td>0.0410</td>
<td>0.0441</td>
<td>0.0245</td>
<td>0.0239</td>
<td>0.0507</td>
<td>0.0816</td>
<td>0.0249</td>
<td>0.0411</td>
<td>0.0958</td>
<td>0.0679</td>
<td>0.0482</td>
<td>0.0418</td>
<td>0.0656</td>
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<tr>
<td>Skewness</td>
<td>-1.86</td>
<td>-1.25</td>
<td>-0.54</td>
<td>0.35</td>
<td>-1.09</td>
<td>0.69</td>
<td>0.71</td>
<td>-1.11</td>
<td>0.76</td>
<td>0.16</td>
<td>0.00</td>
<td>-0.29</td>
<td>0.26</td>
<td>-1.88</td>
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<tr>
<td>Kurtosis</td>
<td>13.00</td>
<td>5.56</td>
<td>3.72</td>
<td>5.49</td>
<td>7.45</td>
<td>3.35</td>
<td>5.77</td>
<td>6.34</td>
<td>7.04</td>
<td>3.94</td>
<td>3.59</td>
<td>5.93</td>
<td>4.47</td>
<td>9.65</td>
</tr>
<tr>
<td>mean beta</td>
<td>0.253</td>
<td>0.363</td>
<td>0.429</td>
<td>0.141</td>
<td>0.157</td>
<td>-0.018</td>
<td>0.828</td>
<td>0.147</td>
<td>0.106</td>
<td>-0.999</td>
<td>0.578</td>
<td>0.180</td>
<td>0.370</td>
<td>1.00</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.1177</td>
<td>0.1138</td>
<td>0.1165</td>
<td>0.0489</td>
<td>0.1138</td>
<td>0.0488</td>
<td>0.1996</td>
<td>0.0774</td>
<td>0.0351</td>
<td>0.3446</td>
<td>0.1407</td>
<td>0.1233</td>
<td>0.0847</td>
<td></td>
</tr>
<tr>
<td>mean co-skewness</td>
<td>0.3330</td>
<td>-0.5901</td>
<td>-0.2444</td>
<td>0.1249</td>
<td>-0.0736</td>
<td>0.2007</td>
<td>-1.3895</td>
<td>0.1807</td>
<td>-0.2919</td>
<td>-0.3645</td>
<td>0.3069</td>
<td>-0.1644</td>
<td>0.0594</td>
<td>-0.0897</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.3077</td>
<td>0.3548</td>
<td>0.1400</td>
<td>0.5582</td>
<td>0.2987</td>
<td>0.2209</td>
<td>0.6370</td>
<td>0.1257</td>
<td>0.1499</td>
<td>0.2042</td>
<td>0.1483</td>
<td>0.3023</td>
<td>0.0423</td>
<td>0.2948</td>
</tr>
<tr>
<td>mean co-kurtosis</td>
<td>1.8156</td>
<td>5.4175</td>
<td>7.8545</td>
<td>1.2149</td>
<td>0.3742</td>
<td>-4.6259</td>
<td>7.8446</td>
<td>5.0790</td>
<td>1.7692</td>
<td>-6.9348</td>
<td>9.8434</td>
<td>2.6956</td>
<td>4.1171</td>
<td>2.7528</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>1.3115</td>
<td>3.1998</td>
<td>4.6592</td>
<td>0.7218</td>
<td>0.2274</td>
<td>2.7447</td>
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<td>1.0537</td>
<td>4.0977</td>
<td>5.8341</td>
<td>2.8643</td>
<td>2.4406</td>
<td>1.5530</td>
</tr>
</tbody>
</table>

Notes: The acronyms of the strategies are reported in Table 1. GI is the GAI general index. MKT is the market return as measured by the S&P500. The conditional beta, co-skewness and co-kurtosis are computed using the multivariate GARCH procedure, as explained in Section 2.1. Std. Dev. stands for standard deviation.
### Table 3 Tests for Threshold VARs

<table>
<thead>
<tr>
<th>GDP growth shock</th>
<th>Wald Statistics</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Threshold value</td>
<td>Sup</td>
<td>Avg</td>
<td>Exp</td>
</tr>
<tr>
<td>beta</td>
<td>0.0042</td>
<td>54.14</td>
<td>30.74</td>
<td>23.78</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.020</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>co-kurtosis</td>
<td>0.2336</td>
<td>87.93</td>
<td>62.61</td>
<td>41.40</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>co-skewness</td>
<td>0.0054</td>
<td>41.24</td>
<td>18.68</td>
<td>18.15</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.040</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>VIX shock</td>
<td>Wald Statistics</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>beta</td>
<td>0.4207</td>
<td>153.20</td>
<td>99.22</td>
<td>72.93</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>co-kurtosis</td>
<td>0.3435</td>
<td>75.47</td>
<td>53.21</td>
<td>34.96</td>
</tr>
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<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>co-skewness</td>
<td>0.3435</td>
<td>85.23</td>
<td>54.06</td>
<td>38.72</td>
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<tr>
<td></td>
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Notes: This table gives the threshold values separating regimes. It also provides the maximum (Sup), average (Avg) and exponential (Exp) Wald statistics associated with the simulations performed over the threshold value (Balke, 2000).
**Figures**

**Figure 1** Co-kurtosis of the hedge fund general index and of some key strategies, 1988-2016

Notes: The acronyms of the strategies are listed in Table 1. GI is the hedge fund GAI general index. MKT is the market return as measured by the S&P500. The conditional co-kurtosis is computed using the multivariate GARCH procedure, as explained in Section 2.1. The periods associated with recessions are shaded.
Figure 2  Beta and co-kurtosis of the hedge fund general index and of key strategies

Notes: The conditional beta and co-kurtosis are computed using the multivariate GARCH procedure, as explained in Section 2.1. The periods associated with recessions are shaded.
Figure 3  Co-skewness of the hedge fund general index and of some key strategies, 1988-2016

Market return, General index, Equity market neutral, Growth

General index, Value index, Futures, Short-sellers

Notes: The conditional co-skewness is computed using the multivariate GARCH procedure, as explained in Section 2.1. The periods associated with recessions are shaded.
Figure 4 Strategies' kurtosis

Notes: Kurtosis is computed using a rolling window of 12 quarters. The periods associated with recessions are shaded.
Figure 5 Strategies' skewness

Notes: Skewness is computed using a rolling window of 12 quarters. The periods associated with recessions are shaded.
Figure 6  Non-linear local projection IRFs of strategies' betas over an expansion period (2013-2016) v/s standard linear VAR

General index

Convertibles

Distressed

Event driven

Equity market neutral

Fixed income

Futures

Growth

Long-short credit
Notes: These plots represent the impulse responses of the strategies' betas to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon of the forecast is fixed at 12 months.
Figure 7 Non-linear local projection IRFs of strategies’ co-kurtosis over an expansion period (2013-2016) v/s standard linear VAR

General index

Equity Market Neutral

Futures

Growth index

Macro

Short-sellers

Notes: These plots represent the impulse responses of the strategies’ co-kurtosis to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon of the forecast is fixed at 12 months.
Figure 8 Non-linear local projection IRFs of strategies’ co-skewness over an expansion period (2013-2016) v/s standard linear VAR
Notes: These plots represent the impulse responses of the strategies’ co-skewness to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon of the forecast is fixed at 12 months.
Figure 9 Non-linear local projection IRFs of strategies' betas over the subprime crisis v/s standard linear VAR
Notes: These plots represent the impulse responses of the strategies’ betas to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon h of the forecast is fixed at 12 months.
Figure 10 Non-linear local projection IRFs of strategies’ co-kurtosis over the subprime crisis v/s standard linear VAR

Notes: These plots represent the impulse responses of the strategies’ co-kurtosis to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon of the forecast is fixed at 12 months.
Figure 11 Non-linear local projection IRFs of strategies' co-skewness over the subprime crisis v/s standard linear VAR
Notes: These plots represent the impulse responses of the strategies' co-skewness to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection, while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon of the forecast is fixed at 12 months.
Figure 12 Cross-sectional dispersions of strategies’ betas, co-kurtosis and co-skewness

Notes: These cross-sectional dispersions are explained in Section 5. The periods associated with recessions are shaded. For more detail on the computation of cross-sectional dispersions, see Racicot and Théoret (2016).
Figure 13 Non-linear local projection IRFs of strategies’ moments cross-sectional dispersions over an expansion period (2013-2016) and over the subprime crisis v/s standard linear VAR

Notes: These plots represent the impulse responses of the strategies’ cross-sectional dispersions to their own shocks and to shocks of the explanatory variables. The outer dotted lines (red lines) embed the 90% confidence interval associated with local projection. The inner dotted line (blue line) is the impulse response function related to the nonlinear local projection while the solid line (pink line) is the impulse response function related to the conventional linear VAR model. The horizon $h$ of the forecast is fixed at 12 months. For more detail on the computation of cross-sectional dispersions, see Racicot and Théoret (2016).
Figure 14 Response of general index beta, co-kurtosis and co-skewness to a GDP growth shock, low and high regimes

Notes: This figure displays the IRFs corresponding to a GDP shock in the low and high regimes using the Balke’s (2000) methodology. These charts are built for positive and negative shocks of two sizes: a one standard deviation (1SD) and two standard deviation (2SD) shocks.
Figure 15 Response of general index beta, co-kurtosis and co-skewness to a VIX shock, low and high regimes

**Low regime**
- Response of general index beta to shock to VIX, low regime
- Response of general index co-kurtosis to shock to VIX, low regime
- Response of the co-skewness of general index to a VIX shock, low regime

**High regime**
- Response of general index beta to shock to VIX, high regime
- Response of the co-kurtosis of the general index to shock to VIX, high regime
- Response of the co-skewness of general index to a VIX shock, high regime

Notes: This figure displays the IRFs corresponding to a VIX shock in the low and high regimes using the Balke’s (2000) methodology. These charts are built for positive and negative shocks of two sizes: a one standard deviation (1SD) and two standard deviation (2SD) shocks.