The impact of macroeconomic and liquidity shocks on hedge fund tail risk

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Abstract
Risk related to return higher moments is usually perceived as substantial in the hedge fund industry. In this article, we relate the return co-skewness and co-kurtosis of hedge fund strategies to macroeconomic and financial shocks. We find that the hedge funds’ return higher moments are quite sensitive to these shocks, and especially during the subprime crisis. Our robust IV-GMM estimation procedure reveals that output shocks are more important for explaining the co-kurtosis of most hedge fund strategies, than shocks related to jumps in the VIX, while we find the opposite for co-skewness. This suggests that hedge funds better monitor their co-kurtosis. Our VAR (vector autoregressive) experiments show that the bulk of the reaction of higher moment risk to shocks occurs during a recession. Furthermore, some directional strategies—like short-sellers and futures’ strategies—successfully monitor shocks and deliver higher returns during periods of crisis. Our results also suggest that illiquidity factors are subject to endogeneity and may be viewed as indicators of return smoothing, especially during crises. Robustness checks also reveal that the practice of return smoothing may lead to a serious underestimation of hedge fund higher moment systematic risk, and especially fat-tail risk.

Keywords: co-skewness; co-kurtosis; hedge funds; diversification; robust IV-GMM; VAR.
JEL classification: C13; C58; G11; G23.

L’impact des chocs macroéconomiques et de liquidité sur le risque extrême de fonds de couverture

Résumé

Mot-clefs: co-skewness; co-kurtosis; fonds de couverture; diversification; IV-GMM robuste; VAR.
Classification JEL: C13; C58; G11; G23.
1. Introduction

In finance, portfolio risk is usually measured by its return standard deviation. This measure was introduced by Markowitz (1952) who recently reiterated his faith in this measure of risk (Markowitz, 2012). Markowitz's theory is developed for portfolios which generate linear payoffs with respect to investors' final wealth. However, for portfolios providing non-linear payoffs\footnote{More precisely, investing in traditional securities entails a linear payoff function—i.e., a linear risk sharing rule. This payoff function may be written as: \( g(x) = w_1 + \alpha X \), where \( w_1 \) is the investor's risk-free wealth at time 1 and \( X \) is the market's portfolio payoff or any other relevant random variable, whereas \( \alpha \) is the investor's exposure to \( X \), a decision variable (Gollier et al., 1995; Franke et al., 1998). Introducing Arrow-Debreu contingent claims—i.e., derivatives—makes the payoff function non-linear. This function may be written as: \( g(x) = w_1 + \alpha X + \theta \left[ \varphi(X) \right] \), where \( \theta \left[ \varphi(X) \right] \) is the investor's payoff for non-linear claims. The payoff \( g(x) \) is convex with respect to \( X \) for investors who buy insurance—i.e., who buy put options, among others. This function is concave for agents who sell insurance (Franke et al., 1998). Agents who buy insurance may be agents with high background risk—i.e., non-insurable risk like risk related to labor income (Franke et al., 1998). Or they may be investors with average return expectations but whose risk tolerance increases more quickly than average with wealth, like pension or endowment funds (Leland, 1980; Gollier, 2001). Finally, investors buying insurance may be investors whose expectations of returns are more optimistic than average and who are in search of high \( \alpha \) like hedge funds (Leland, 1980; Gollier, 2001).} with respect to their underlying assets, the return standard deviation stands as an imperfect measure of risk. Indeed, even if investments in hedge funds are generally considered riskier than traditional investments, the standard deviation of hedge fund strategies' return is often lower (Boyson et al., 2010; Hübner et al., 2011). We thus have to rely on return higher moments to gauge the asymmetric payoffs delivered by hedge funds (Fung and Hsieh, 1997, 2001, 2004; Cumming et al., 2014).

In this paper, our main objective is to develop new empirical measures of tail risk for hedge fund strategies and to relate them to indicators of macroeconomic, financial, and illiquidity risk in a dynamic setting which accounts for asymmetries in the behavior of hedge funds over the business cycle. Our measures of tail risk—i.e., co-skewness and co-kurtosis—are based upon the four-moment CAPM. They represent systematic tail risk since, like the beta, they account for portfolio diversification. To the best of our knowledge, we are the first to rely on these tail risk indicators to conduct a dynamic enquiry on the risk behavior of hedge funds over the business cycle.

Our second objective aims at studying the endogeneity of hedge fund risk as measured by the higher moments of strategies' returns. Indeed, one problem when dealing with risk borne by hedge funds is that this risk may be considered as endogenous. Even if hedge funds may suffer from financial and macroeconomic shocks, they can also monitor (manage) risk by resorting to many tools which are less used by other more regulated financial institutions—e.g., mutual funds—like hedging and especially short selling. For instance, hedge funds may decrease the risk of their payoffs by buying positively skewed payoffs and
increase their risk by buying “fat-tailed” payoffs—i.e., payoffs with higher kurtosis (Hübner et al., 2011). Hedge funds can also follow the trend of the market, which obviously leads to asymmetric (non-linear) payoffs—market-timing being originally accounted for by the square of the market return or by a dummy variable (Treynor and Mazuy, 1966; Henriksson and Merton, 1981; Henriksson, 1984; Gollier et al., 1995; Asness et al., 2001; Fung and Hsieh, 2001; Bali et al., 2014). Measures of macroeconomic and illiquidity shocks, which are the ingredients of our empirical models, are also endogenous. Indeed, measures of macroeconomic shocks are generated variables, and thus endogenous variables (Pagan, 1984, 1986; Pagan and Ullah, 1988), and illiquidity shocks are also endogenous because they depend on the behavior of market makers (Brunnermeier and Sannikov, 2014; Adrian et al., 2017). We account for these endogeneity issues by relying on GMM with our robust instruments (Racicot and Théoret, 2014; Racicot et al., 2018).

To implement our research program, we develop macroeconomic and financial indicators of shocks similar to Beaudry et al. (2001), Baum et al. (2002, 2004, 2009), Quagliariello (2009), and Bali et al. (2014). In contrast to most of these researchers who rely on the GARCH procedure\(^2\) (Bollerslev et al., 1986), we build EGARCH measures\(^3\) of macroeconomic and financial shocks. Indeed, there are important asymmetries in the behavior of shocks which are mostly observed during bad times. The reaction of hedge funds to these shocks is also more severe during bad times and much smoother during good times. Moreover, since the empirical measures we develop for shocks are generated variables, we must account for the biases embedded in the OLS estimations of our co-moment equations (Pagan, 1984, 1986; Pagan and Ullah, 1988; Calmès and Théoret, 2014). To do so, we resort to the generalized method of moments (GMM). We still rely on our robust instruments to implement this IV method (Fuller, 1987; Lewbel, 1997; Racicot and Théoret, 2014; Racicot et al., 2018). Moreover, to evaluate the extent of the biases, we compare our OLS results with GMM ones.

Our second contribution is to apply a new methodology to study the dynamics of co-skewness and co-kurtosis over the business cycle which accounts for macroeconomic and liquidity risk and uncertainty. In this regard, we resort to a VAR analysis which distinguishes expansions from recessions (or crises) in order to study asymmetries in the behavior of hedge funds. Other studies which target the dynamics of hedge fund rely on cross-section and

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\(^2\) Bali et al. (2014) rely on a multivariate-TARCH procedure to implement their experiments.

\(^3\) The EGARCH procedure is due to Nelson (1991). For the building of macroeconomic shocks using this procedure, see Calmès and Théoret (2014) and Racicot and Théoret (2016).
quantile analyses (Agarwal et al., 2017b; Stafylas et al., 2017; González and Jareño, 2018). In this respect, low return quantiles are associated with recessions (or crises) and high return quantiles are related to expansions. We think that it is preferable to rely on time series analyses—especially VARs—to perform business cycle studies.

Our third contribution is to show how return common factors—i.e. the size factor (SMB), the value factor (HML)\(^4\) and the straddle lookback factor\(^5\)—react to shocks and thus might embed the co-skewness and co-kurtosis of hedge fund return distributions. Finally, as a robustness check, we transpose the practice of return smoothing to our new empirical measures of co-skewness and co-kurtosis by using a moving average process (MA(k)) to correct co-skewness and co-kurtosis for return smoothing (Getmansky et al., 2004).

We document six major findings. First, aside factors related to financial market volatility, macroeconomic factors are important for explaining hedge fund higher moments even if they are often neglected in the finance literature. Co-skewness of most strategies is more sensitive to VIX than to the production shock while the opposite holds for co-kurtosis. In this regard, strategies seem to better track their co-kurtosis which is less sensitive to VIX—a more predictable indicator—than co-skewness. This is especially the case during crises. Co-kurtosis estimations are also more subject to endogeneity than co-skewness. This is another evidence that hedge funds monitor their co-kurtosis during bad times at the expense of co-skewness, co-kurtosis risk being a more important issue during crises than co-skewness one.

Second, for most strategies, illiquidity factors—i.e., the Pastor and Stambaugh (PS, 2003) illiquidity factor and the Amihud (2002)—may be viewed as indicators of return smoothing in the sense that an increase in illiquidity results in a decrease of tail risk in the hedge fund industry. However, as revealed by the VAR, this practice is not persistent. Third, there exists obvious asymmetries in the behavior of hedge fund strategies. Indeed, uncertainty factors are mainly at play in recessions for explaining co-skewness and co-kurtosis and are usually not significant in expansions.

Fourth, short-sellers and futures behave differently from other strategies—especially in recessions. This result is partly related to the positive correlation of their returns with lookback straddles ones. These two strategies are thus a good hedge for investors during crises. Fifth, the VIX and PS are two different indicators of illiquidity, PS being negatively and

\(^4\) SMB and HML being factors in the Fama and French (1992, 1993) asset pricing model.

\(^5\) Built using PCA over the five Fung and Hsieh’s (2001) lookback straddles.
weakly correlated with VIX. PS becomes more unstable when VIX increases. In contrast, the Amihud ratio is quite positively correlated with VIX. Illiquidity has thus many dimensions.

Finally, return smoothing may substantially understate hedge fund co-skewness and co-kurtosis. Our results show that return smoothing impacts more co-kurtosis than co-skewness. Hedge fund general index co-kurtosis may be even higher than the one of the market return after unsmoothing while it is much lower on a smoothed basis. Moreover, market return co-skewness has a more favorable profile during expansions.

This article is organized as follows. Section 2 provides the methodology used in our paper. In this section, we present our indicators of return co-skewness and co-kurtosis, the method to build macroeconomic and financial shocks, and our empirical model. We also address some empirical issues like the asymmetry of the squared innovations of our shock variables (i.e., EGARCH) and the endogeneity associated with generated variables (shocks) which is tackled by our robust IV-GMM. Section 3 is an analysis of our database. In this section, we are particularly interested in the stylized facts associated with strategies’ co-skewness and co-kurtosis. Section 4 discusses our empirical results while Section 5 concludes.

2. Methodology

2.1 The concepts of co-skewness and co-kurtosis

The introduction of return co-moments in asset pricing models originates from Samuelson (1970) and Rubinstein (1973). Later, Kraus and Litzenberger (1976) developed the three-moment CAPM which introduces return co-skewness as an additional explanatory variable in the standard Sharpe-Lintner-Mossin CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966) while Fang and Lai (1997), among others, added co-kurtosis as another higher co-moment in the CAPM model.

The idea behind the addition of co-skewness and co-kurtosis to asset pricing models complements the presence of beta. The beta measure accounts for risk associated with the return second moment—i.e., the return standard deviation. But since part of the risk related to standard deviation is diversifiable, only non-diversifiable risk as measured by the beta is priced by the market. Similarly, as investors dislike second moment risk, they dislike risk associated with the return fourth moment, i.e., kurtosis, and have a preference for risk related

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6 For a survey of the multi-moment CAPM, see Jurczenko and Maillet (2006).
to the third moment, i.e., positive skewness. But, in accordance with standard finance theory, only the non-diversifiable component of skewness and kurtosis is priced by the market. Consequently, only co-skewness and co-kurtosis impact security returns.

More precisely, the various even moments of the distribution of a random variable gauge many dimensions of dispersion and thus correspond to uncertainty, which provides disutility to risk-averse agents. In other respects, odd higher moments are associated with different dimensions of localization with respect to the mean. Agents try to maximize these odd moments, which thus provide them a positive utility (Desmoulins-Lebeault, 2006).

To cast co-skewness and co-kurtosis as state variables in an intertemporal optimal portfolio selection framework, we can rely on the well-known Euler first-order condition (Cochrane, 2005), i.e.,

\[ p_{t+1} = E\left[ m_{t+1}x_{t+1} \middle| \Omega_t \right] \quad (1) \]

where \( p_{t+1} \) is the price of an asset at time \( t+1 \); \( m_{t+1} \) is the representative agent’s intertemporal marginal rate of substitution (discount factor); \( x_{t+1} \) is the payoff of the asset at time \( t+1 \), and \( \Omega_t \) is the information set available to the representative agent at the time of his portfolio decision.

We can express equation (1) in terms of return by dividing both sides by \( p_{t+1} \):

\[ E\left[ m_{t+1}R_{t+1} \middle| \Omega_t \right] = 1 \quad (2) \]

where \( R_{t+1} \) is the gross rate of return of the asset at time \( t+1 \).

If the assumptions of the CAPM are satisfied—i.e., if the return distribution is Gaussian or completely characterized by its first two moments, or if the representative agent’s utility function is quadratic—\( m_{t+1} \) can be written as a linear function of the market return (Desmoulins-Lebeault, 2006):

\[ m_{t+1} = a + b_tR_{mt+1} \quad (3) \]

where \( R_{mt+1} \) is the market return.

However, if the assumptions associated with the CAPM do not hold—especially if the return distribution is non-Gaussian—the relation between \( m_{t+1} \) and \( R_{mt+1} \) is nonlinear. Assume that \( m_{t+1} \) may be written as the following third-order polynomial of \( R_{mt+1} \):

\[ m_{t+1} = a + b_tR_{mt+1} + c_tR_{mt+1}^2 + d_tR_{mt+1}^3 \quad (4) \]

Remark 7. In fact, investors have a preference for odd-moments, as positive expected return and positive skewness, and dislike the even moments, as return standard deviation and kurtosis (Scott and Hovarth, 1980).
We then obtain the fourth-moment CAPM asset pricing model that may be expressed as follows (Fang and Lai, 1997):

\[
E(R_i) - r_f = \beta_1 \text{Cov}(R_m, R_i) + \beta_2 \text{Cov}(R_m^2, R_i) + \beta_3 \text{Cov}(R_m^3, R_i)
\]  

(5)

where \(E(R_i)\) is the expected value of return \(i\), and \(r_f\) is the risk-free rate. In equation (5), \(\text{Cov}(R_m^2, R_i)\) is co-skewness and \(\text{Cov}(R_m^3, R_i)\) is co-kurtosis. To build our co-skewness and co-kurtosis time series for each strategy, we compute a moving average of the respective co-variances over twelve months.

Insert Figure 1 and Table 1 here

2.2 The construction of shocks time series

Table 1 provides the list of the shocks variables used in this study to explain the behavior of co-skewness and co-kurtosis. Two variables are non-generated variables, in the sense that they are not built using GARCH procedures: the implied volatility of the S&P500—i.e., the VIX—, and the credit spread. The VIX is a good indicator of tensions on financial markets—i.e., market risk and investors’ fears—while the credit spread is an indicator of credit risk. As evidenced by Figure 1, the VIX tends to jump during a crisis. In our sample, its increase was important during the Asian-Russian-LTCM crisis (1997-1998) and especially during the subprime crisis (2007-2009). The VIX behavior is much smoother otherwise. Its plot is thus quite asymmetric, an important dimension in the context of our study since, as shown later, the behavior of co-skewness and co-kurtosis is also very asymmetric dependent on the phase of the business cycle. In other respects, the credit spread is an important countercyclical variable (Figure 1). It increases during recessions while it is much lower during expansions. For instance, it jumped to a very high level during the subprime crisis, the dominant factor during this crisis being credit risk, or risk related to subprime (risky) mortgages.

The other variables accounting for macroeconomic and financial shocks which are listed in Table 1 are generated variables (Pagan 1984, 1986). The construction of the conditional variances of the industrial production growth and inflation—\(cv_{gprod}\) and \(cv_{inf}\) respectively—is explained in detail in Baum et al. (2002, 2004, 2009). These variables are in fact the conditional variances of an ARMA model applied to the industrial production growth and inflation. However, in contrast to Baum et al. (2002, 2004, 2009) who rely on a GARCH procedure (Bollerslev, 1986) to model the conditional variance, we resort to an EGARCH
procedure\textsuperscript{8} (Nelson, 1991) to do so. Indeed, there are important asymmetries in the squared innovations of our ARMA models which must be tackled by an EGARCH procedure. We also build four other conditional variances for the three-month Treasury bill ($cv_{tb}$), the growth of the commodities prices ($cv_{gmp}$), the return on the S&P500 ($cv_{rsp500}$) and the return on the U.S. exchange rate ($cv_{rexch}$). The construction of these conditional variances obeys to the same procedure followed for $cv_{gprod}$ and $cv_{inf}$. Further details on the construction of our generated variables are provided in Table 1.

Figure 1 shows that shocks stemming from industrial production growth—as measured by $cv_{gprod}$—occur during periods of recessions—i.e, when the output gap drops sharply. The output shock was especially important during the subprime crisis. Outside crises, $cv_{gprod}$ is low and quite stable. There is thus an important asymmetry in the behavior of $cv_{gprod}$ according to the phases of the business cycle. In other respects, the shocks to inflation—as measured by $cv_{inf}$—are more or less related to the output gap. There are usually inflation shocks during recessions, this shock having been quite severe during the subprime crisis (deflation). However, inflation shocks may also occur outside recessions, like the one observed in 2005 which led to a tightening of the US monetary policy. Note that the inflation shock may be associated either with a surging consumer price index or a drop in this index. Therefore, the $cv_{inf}$ series contains information which is not embedded in the $cv_{gprod}$ one and may thus be of a great help in the estimation exercise.

Similarly to $cv_{inf}$, the shocks emanating from short-term interest rates—as measured by $cv_{tb}$—may occur outside recessions. For instance, there was an important interest rate shock during the Asian-Russian LTCM crisis (Figure 1). Finally, the shocks associated with the S&P500 return tend to be synchronized to the shocks measured by $cv_{gprod}$.

To account for illiquidity risk, we also add the the Pástor and Stambaugh (PS, 2003) illiquidity risk factor in our canonical higher moment model. The $PS$ illiquidity factor is an example of what might be considered a generated variable because it is a parameter obtained from a regression, in this case relating stock return to its trading volume. An increase in $PS$ is associated with a rise in market illiquidity. However, as shown in Figure 1, even if $PS$ peaks during periods of crises or recessions, it also becomes much more volatile. Illiquidity is thus associated with a greater instability of this indicator.

\textsuperscript{8}The EGARCH procedure is explained in the following section. To account for asymmetries, Bali et al. (2014) rely instead on a TARCH procedure, as discussed later.
Table 2 displays the correlation between our shock variables. Most of the shock indicators have a high correlation with $cv_{gprod}$. The credit spread is the variable which is most correlated to $cv_{gprod}$ (0.74), followed by $cv_{gmp}$ (0.67), $cv_{tbw}$ (0.60) and $cv_{inf}$ (0.57). The traditional macroeconomic variables—i.e., production growth, inflation and the interest rate—thus feature shocks which are strongly correlated. Shocks on the S&P500 return are also quite correlated with the shocks associated with the standard macroeconomic variables. For instance, the correlation of $cv_{rsp500}$ with $cv_{gprod}$ is equal to 0.51. The $VIX$ variable also entertains a high correlation with $cv_{rsp500}$ (0.61), the credit spread (0.60) and $cv_{gprod}$ (0.52). In this respect, the high correlation between $cv_{rsp500}$ and the $VIX$ supports Black’s (1976) leverage effect, which relates the decrease in stock returns to jumps in market volatility. Note that the credit spread variable displays an especially high correlation with the shocks associated with macroeconomic variables. Its correlation with $cv_{gprod}$, $cv_{gmp}$, $cv_{rsp500}$ and $cv_{inf}$ is 0.81, 0.75, 0.63 and 0.59, respectively. Finally, the illiquidity variable ($PS$) is only correlated significantly with the $VIX$, the coefficient being -0.15. This negative correlation is surprising since the $VIX$ is also often viewed as an indicator of illiquidity (Brunnermeier and Pedersen, 2009; Adrian et al., 2017; Malkhosov et al., 2017). For instance, similarly to the realized volatility, the $VIX$ is quite correlated with the high frequency reversal returns, which tend to peak during recessions or crises (Adrian et al., 2017). However, as explained previously, the $PS$ illiquidity factor display higher fluctuations during financial crises and does not trend upwards like the $VIX$ (Figure 1). This may explain the negative (low) correlation between the $VIX$ and $PS$. $PS$ thus measures illiquidity dimensions which differ from the $VIX$.

2.3 The model

Co-skewness and co-kurtosis are related to extreme events associated with the tails of the return distribution—especially the left tail associated with fat-tail risk (Merton, 1992). In this respect, as shown later, co-skewness and co-kurtosis are subject to jumps during periods of crisis but are low and quite stable otherwise. A simple model relating co-skewness

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9As mentioned by Adrian et al. (2017), market liquidity is the cost of exchanging assets for cash. This cost has many dimensions.
and co-kurtosis to macroeconomic and financial shocks thus seems particularly appropriate. This model reads as follows

\[
i = 1 \text{ to } k \quad y_{it} = \alpha_i + \beta^i X_{it} + \theta_i y_{it-1} + \varepsilon_{it} \quad (6)
\]

where \( y_{it} \) is co-skewness or co-kurtosis of hedge fund strategy \( i^{10} \); \( X_{it} \) stands for the vector of shocks variables which are listed in Table 1; \( y_{it-1} \) is the lagged value of \( y_{it} \) accounting for autocorrelation and the delayed adjustment of \( y_{it} \) to its target value, and \( \varepsilon_{it} \) is the innovation.

In this kind of model, return smoothing may be viewed as a delay (partial adjustment) in the adjustment of hedge fund risk measures to their effective (unobserved) values.

To make the estimated coefficients comparable, we transform them into elasticities (Baum et al., 2002, 2004, 2009; Calmès and Théoret, 2014). The elasticity of \( y \) with respect to \( x \) is computed using the following formula\(^{11}\): 

\[
\frac{\partial y}{\partial x} \times \frac{\bar{x}}{\bar{y}}, \quad \text{where} \quad \text{coef} \text{ is the estimated coefficient of } x, \text{ and } \bar{x} \text{ and } \bar{y} \text{ are, respectively, the average values of } x_t \text{ and } y_t \text{ computed over the whole sample period}\(^ {12}\). The elasticity is thus calculated at the point of the mean of each independent variable and may thus be called “elasticity at means”\(^ {13}\).

### 2.4 Estimation issues

#### 2.4.1 The EGARCH procedure

To estimate the conditional variances of our macroeconomic and financial variables, we rely on a GARCH(1,1) and more frequently on an EGARCH (1,1). The equation structure used is the following:

1. a mean equation, which is an ARMA \((p,q)\) specification of the (stationary) macroeconomic times series used to measure macroeconomic uncertainty. It thus takes the following form:

\[
b(\ell) y_t = \gamma + c(\ell) \varepsilon_t \quad (7)
\]

where the lag operators \( b(\ell) \) and \( c(\ell) \) are equal to: 

\[
b(\ell) = \sum_{i=0}^{p} b_i \ell^i, \quad b_0 = 1, \quad b_i \neq 0 = -\beta_i
\]

\(^{10}\)There are \( k \) strategies.

\(^{11}\)By definition, elasticity is equal to: 

\[
\frac{\partial y}{\partial x} = \frac{\gamma_x}{\gamma} + \frac{\beta y_x}{\gamma} - \frac{x}{\gamma}, \quad \text{and from (6)} \quad \frac{\gamma_x}{\gamma} - \beta = \frac{\gamma_x}{\gamma} - \beta \times \frac{x}{\gamma}.
\]

\(^{12}\)For more detail on this formula, see Pindyck and Rubinfeld (1998), p. 99.

\(^{13}\)The mean values of \( x \) and \( y \)—i.e., the scaling factors—are considered as constants when computing the elasticity. Thus the p-value (and t-statistic) of the elasticity is the same as the p-value (t-statistic) of the coefficient used to compute the elasticity.
, and $c(t) = \sum_{j=0}^{q} c_j t^j$.

(ii) a variance equation, which may be a GARCH(1,1) (Bollerslev, 1986)—i.e.,

$$
\sigma_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 \sigma_{t-1}^2
$$

or an EGARCH(1,1) (Nelson, 1991)—i.e.,

$$
\ln(\sigma_t^2) = \eta_1 + \eta_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \eta_3 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \eta_4 \ln(\sigma_{t-1}^2) - \frac{2}{\sqrt{\pi}}
$$

According to Eq. (8), the conditional variance of the innovation is related to the lagged squared innovation and the lagged conditional variance. The sum of the coefficients $\phi_1$ and $\phi_2$ is a measure of persistence. Eq. (9) adds an asymmetrical effect in the conditional variance model. This effect depends on the sign of $\varepsilon_{t-1}$. If $\varepsilon_{t-1} > 0$, the total effect of $\varepsilon_{t-1}$ on the log of the conditional variance can be measured by $(\eta_3 + \eta_2) \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right|$, while if $\varepsilon_{t-1} < 0$, it can be measured by $(\eta_3 - \eta_2) \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right|$. Thus, the asymmetric leverage effect can be tested with the coefficient $\eta_2$. If $\eta_2 < 0$, the asymmetrical effect is higher when $\varepsilon_{t-1} < 0$. This is the Black's (1976) leverage effect, whereby the volatility of the returns on a stock is higher when the price of this stock trends downward. In contrast, if $\eta_2 > 0$, the asymmetrical effect is higher when $\varepsilon_{t-1} > 0$. Further, volatility persistence increases with $\eta_4$ in the EGARCH model.

To tackle the asymmetries associated with uncertainty shocks, Bali et al. (2014) rely on a multivariate-TARCH, a method which also accounts for the co-movements between the economic and financial time series used to compute the shocks. We thus also experimented with this model of conditional variance, using a set of variables similar to Bali et al. (2014). We compared this model with standard (univariate) TARCH and EGARCH models. However, in the context of our study, the multivariate-TARCH did not perform better than an EGARCH model, so we resorted to the latter to compute our shocks. The reasons which explain our choice are the following ones:

(i) the profiles of the shocks estimated with the multivariate-TARCH and
standard EGARCH models, or with the standard TARCH model, are essentially the same, the correlation between the shocks computed with the three conditional variance models being over 0.90. Moreover, the results of the multivariate-TARCH are not very sensitive to the set of variables used to build the system;

(ii) the differences observed between the three measures of conditional variance do not depend so much on the cleavage between an univariate or multivariate approach *per se* than on the kind of GARCH procedure used to tackle the asymmetries—i.e., a TARCH versus an EGARCH model. In our framework, the TARCH model tends to exaggerate the spikes of the shock series, which crunches the other data of the plots. To facilitate the estimations of our models relating systematic risk embedded in return higher moments to shocks, the spikes related to the TARCH procedure have to be winsorized, which is not the case with the EGARCH model that provides smaller spikes.

Overall, to compute the shock series used in this article, we retain the standard EGARCH model. Note also that the number of estimated parameters increases exponentially with the number of variables used to run the multivariate-TARCH, and the degrees of freedom associated with the estimation decreases at the same pace. When using a sample of restricted size as ours, it seems preferable to resort to a more parsimonious model of conditional variance. Finally, since we have no formal structural model linking our macroeconomic and financial variables, a multivariate approach may create spurious correlations or co-movements between variables, which introduces noise in the estimation of the parameters.

### 2.4.2 Endogeneity issues

In this paper, we first estimate equation (6) by OLS, the standard method in econometric work. However, the conditional variances of our macroeconomic and financial variables—our measures of shocks—are generated variables—i.e., potentially noisy proxies for their associated unobservable regressors (Pagan 1984, 1986; Beaudry et al., 2001; Baum et al., 2002, 2004, and 2009; Calmès and Théoret, 2014). As noted before, the illiquidity indicator $PS$ is also an endogenous variable. First, it is a generated variable. Second, it is

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16 In the absence of winsorization, the estimation of the model parameters is dominated by the outliers.

17 This endogeneity issue is not accounted for in Bali et al. (2014). They do not instrument their shocks variables to remove the biases created by generated variables like shocks computed using conditional variance models.
related to the behavior of the market makers (Brunnermeier and Sannikov, 2014; Adrian, et al., 2017).

Relying on ordinary least-squares (OLS) or simple maximum likelihood estimation (MLE) in the presence of generated variables lead to invalid t-tests (Pagan 1984, 1986) and may even result in estimator inconsistency (Pagan and Ullah, 1988). To check the robustness of our OLS results, we thus also estimate equation (6) by our robust IV-GMM in which we first regress the generated variables on instruments. These instruments include the predetermined variables and the higher moments of the models’ explanatory variables (Fuller, 1987; Lewbel, 1997; Racicot and Théoret, 2014; Racicot et al., 2018). Note that GMM display asymptotic properties with respect to the correction of heteroskedasticity and autocorrelation to weight the instruments obtained with the Generalized least squares estimation method (GLS). When using GMM, we give up some efficiency gain in order to avoid the complete specification of the nature of the autocorrelation or heteroskedasticity of the innovation and the data generating process (DGP) of the measurements errors (Hansen, 1982). This is also a great advantage over GLS.

The GMM estimator may be written as follows (Racicot and Théoret, 2001):

$$\hat{\beta} = \arg \min_{\beta} \left\{ n^{-1} \left[ Z^T (y - X\hat{\beta}) \right]^T W n^{-1} \left[ Z^T (y - X\hat{\beta}) \right] \right\}$$

(10)

In Eq.(10), $Z$ is the matrix of instrumental variables; $y$ is the dependent variable; $X$ is the matrix of the explanatory variables, and $W$ is a weighting matrix. To implement the GMM, we resort to an innovative set of instruments defined as:

$$d_{it} = x_{it} - \hat{x}_{it}$$

(11)

with $\hat{x}_{it}$ the predicted value of $x_{it}$.

These instruments—called the $d$ instruments or distance instruments—may be considered as filtered versions of the endogenous variables. We thus implement a distance metrics to compute our instruments. The $d_{it}$ series removes some of the nonlinearities embedded in the $x_{it}$. It is thus a smooth mapping of the $x_{it}$ which might be regarded as a proxy for its long-term expected value, the relevant variables in the asset pricing models being theoretically defined on the explanatory variables expected values. To compute the $\hat{x}_{it}$ in (11), we perform the following regression using the $z$ (cumulant) instruments:

$$x_{it} = \hat{y}_0 + Z\hat{\phi} + \zeta_{it} = \hat{x}_{it} + \zeta_{it}$$

(12)
The computation of the $z$ instruments is based on our previous works (e.g., Racicot and Théoret, 2014). They are based on the cumulants of the explanatory variables $X$. More specifically, the $z$ instruments are a weighting of Durbin and Pal’s estimators defined for models embedding errors in variables. Finally, our new version of GMM defined on $d$ instruments—named GMM-$d$—obtains:

$$
\arg \min_{\hat{\beta}} \left\{ n^{-1} \left[ d^T (y - X\hat{\beta}) \right]^T W n^{-1} \left[ d^T (y - X\hat{\beta}) \right] \right\} 
$$

(13)

3. Data

3.1 Data sources

The hedge fund strategies’ returns are taken from the database managed by Greenwich Alternative Investment (GAI). GAI manages one of the oldest hedge fund databases, containing more than 13,500 records of hedge funds as of March 2010. Returns provided by the database are net of fees. The survivorship bias is accounted for in this database, as index returns for periods since 1994 include the defunct funds. The dataset runs from January 1995 to September 2012, for a total of 213 observations. In addition to the weighted composite return, the database includes 9 return series of well-known hedge fund strategies reported in Table 3. Data for U.S. macroeconomic and financial variables are drawn from the FRED database, which is managed by the Federal Reserve Bank of St-Louis.

3.2 Hedge fund returns descriptive statistics

Table 3 reports the descriptive statistics of our hedge fund database. There is some heterogeneity in the historical returns and risk characteristics of hedge fund strategies. For instance, the monthly mean returns range from a low of -0.07% for the short sellers$^{18}$ to a high of 1.07% for the value index, whereas the return standard deviation ranges from a low of 1.39% for the market neutral group to a high of 5.83% for the short sellers. A hedge fund’s beta is generally low and the average beta computed over all strategies is equal to 0.17. Two strategies display a negative beta: short sellers (-0.91) and futures (-0.08)$^{19}$. The strategy

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$^{18}$ Note that the negative return delivered by short-sellers should not be viewed as abnormal or excessively low. For example, in the real or physical universe—as opposed to the risk neutral or forward risk neutral universe—if the real world expected required return is 16%, the expected required return of a long put is close to -50% as opposed to 40% for a long call in Hull’s (2012) example.

$^{19}$ Selling short may thus be a dominant strategy for futures.
with the highest positive beta is growth (0.69) while the strategy with the lowest positive beta is, as expected, equity market neutral (0.08).

We can classify hedge fund strategies in two main categories according to the value of their beta: directional and non-directional strategies. Some strategies are directional in the sense that they are more exposed to the fluctuations of the overall stock market. They tend to have a higher beta than the average strategies. In this group, we may include the growth (0.69), value index (0.53), long-short (0.49), macro (0.21), futures (-0.08) and short-sellers’ (-0.91) strategies. Note that the futures strategy displays a low beta but is usually considered as directional. Strategies with the highest beta are usually the ones which display the highest adjusted \( R^2 \) in standard multifactor asset pricing models such as the Fama and French (1992, 1993) model. We classify the other strategies appearing in Table 3 as non-directional strategies—i.e., distressed securities, event-driven and equity market neutral. These strategies are often involved in arbitrage activities.

In Table 3, we also divide our sample period into three subperiods to compute the strategies’ mean return: (i) the period before the subprime crisis—i.e., from January 1995 to May 2007; (ii) the subprime crisis period as such, which extends from June 2007 to December 2009; (iii) the period after the crisis, which is comprised between January 2010 to September 2012 in our sample. The first subperiod is characterized by very high returns in the hedge fund industry. It is also associated with the period of the “alpha puzzle” (Racicot and Théoret, 2013). During this subperiod, the mean return approximated 0.98% monthly which is equivalent to an annual return of about 12%, a quite high return. The short-sellers and macro strategies underperformed, with monthly returns equal to -0.16% and 0.57%, respectively. All the other strategies displayed a monthly return higher than 1% during this subperiod. The period corresponding to the subprime crisis is especially interesting. During this subperiod of depressed returns on financial markets, two strategies displayed a very high monthly return: short-sellers (0.94%) and futures (0.90%). As evidenced in our empirical work, these two strategies usually perform very well in period of crises both in terms of return and risk. The macro strategy also displayed a relatively good monthly return (0.30%) during the subprime crisis. Even if the mean return of this strategy is lower than the one of the weighted composite index (general index) over the whole sample period, it is much less volatile. Finally,

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20 Connor and Lasarte (2005) distinguish two broad categories of hedge fund strategies: the market neutral and directional.

21 Note that Connor and Lasarte (2005) classify the value index strategy in the arbitrage or non-directional category.

22 See: Greenwich Alternative Investment, Greenwich Global Hedge Fund Index Construction Methodology.

23 In contrast, the short-sellers strategy usually displays a negative return in good times while the futures strategy continues to perform quite well.
the recovery of strategies’ returns was very slow after the subprime crisis, the mean monthly return of the weighted composite index having been only 0.26% from January 2010 to September 2012 versus 0.90% over the whole sample period.

Insert Figure 2 here

The standard deviation of the GAI weighted return (henceforth called the general index) is less than the S&P500 return over our sample period, the respective levels being 2.18% and 4.59% (Table 3). In fact, the standard deviation of the return of the general index seems to decline through time, which is not the case for the return of the S&P500 (Figure 2). More importantly, the standard deviation of the index return increased less during the subprime crisis than during the tech bubble, while the standard deviation of the S&P500 return increased much more during the subprime crisis. This is an evidence of a decline of procyclicality in the hedge fund sector. Furthermore, the hedge fund general return index co-moves less (negatively) with the VIX, a measure of volatility on financial markets (Figure 2).

At 0.21, the mean skewness of hedge fund strategies is positive in our sample. Some strategies display negative skewness over the sample period—i.e., value index, distressed and event driven. Returns of directional strategies tend to display a positive skewness. This contrasts with the market portfolio which is negatively skewed. Note that our results are more or less in line with Chan et al. (2007) and Heuson and Hutchinson (2011) who find that most hedge fund strategies show negative skewness, what they consider as an indication of tail risk. However, the skewness statistic is very sensitive to few outliers and is thus not a very reliable measure of tail risk. Our time-varying measure of co-skewness is more representative of hedge fund asymmetric risk than total skewness per se. However, a more straightforward measure of tail risk is kurtosis. Most hedge funds display a higher kurtosis than the market return. In our sample, kurtosis ranges from 3.38 (futures) to 8.77 (equity market neutral). The mean kurtosis amounts to 6.06 for all strategies while it is equal to 3.84 for the market return. This suggests that hedge funds are more exposed to extreme risks than the market portfolio. However, we retain the co-kurtosis as a measure of extreme risk because the diversifiable component of kurtosis, which is removed from co-kurtosis, is not priced by the market.

Finally, Table 3 provides the autoregressive coefficients of order 1 for our hedge funds’ strategies. These coefficients are obtained by regressing strategies’ returns on their first lag. The higher the coefficients, the more we can suspect illiquidity in the hedge funds’ assets under management or return smoothing by hedge funds’ managers (Getmansky et al.,
2004; Chan et al., 2007; Brown et al., 2012; Bali et al., 2014). We compute these coefficients over the whole sample period but also during the subprime crisis, return smoothing or illiquidity being eventually greater in time of crises. While there is no obvious autocorrelation of order one (AR(1)) for the market return over the whole sample period, this is not the case for hedge funds. The AR(1) coefficient associated with the general index is equal to 0.25, significant at the 5% level. Some strategies—i.e., distressed and even driven—display especially high autoregressive coefficients, while other ones—i.e., macro, futures and short-sellers—display no evidence of first-order autocorrelation. Moreover, the strategies plagued with autocorrelation over the whole sample period have much higher AR(1) coefficients during the subprime crisis. For instance, the coefficient of the general index jumps to 0.40 during the subprime crisis while the one of the distressed securities spikes at 0.58. Note that strategies not affected by autocorrelation during the whole sample period—i.e, macro, futures and short-sellers—are also immune to significant autocorrelation during the subprime crisis. Surprisingly, the AR(1) coefficient of the market return becomes significant during the subprime crisis and it is relatively high at 0.38, which suggests that liquidity decreased substantially on financial markets during the subprime crisis or/and that the managers of the firms included in the market index were involved in income smoothing (Ronen and Yaari, 2008). For instance, the reaction of stock returns should be smoother (more autocorrelation) if firms’ managers are involved in income smoothing. This is a phenomenon related to the information asymmetries between firms’ managers and investors24. We will come back on the repercussions of autocorrelated returns on measures of systematic risk in the empirical section, since return smoothing—associated with the level of the strategies’ AR(1) coefficients—may lead to a serious underestimation of hedge funds’ systematic risk (Getmansky et al., 2004; Chan et al., 2007; Brown et al., 2012; Bali et al., 2014).

3.3 Stylized facts about hedge fund strategies’ co-skewness and co-kurtosis

Figure 3 provides the behavior of the co-skewness and co-kurtosis of the hedge fund general index over our sample period. The output gap is also plotted in our figures. Note that the co-skewness tends to deteriorate25 during recessions or financial crises. Systematic risk related to the asymmetry of the return distribution thus increases during bad times. This kind

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24 The Enron’s affair is a good example of this problematic.
25 In the sense that it becomes more negative. Recall that risk-averse investors prefer positive to negative skewness.
of risk was particularly acute during the Asian-Russian-LTCM crisis and especially during the subprime crisis. Risk related to co-skewness tends to decrease during expansion. In other respects, the general index co-kurtosis is much more stable than its co-skewness during expansions. But it jumped during the Asian-Russian-LTCM crisis and during the subprime crisis. Extreme risks thus increase substantially during crises in the hedge fund industry.

Figure 4 compares the plots of the general index co-skewness and co-kurtosis to the corresponding ones of the market return (S&P500). The market return co-skewness is more volatile than the one of the general index. Surprisingly, it deteriorated much more during crises, and especially during the Asian-Russian-LTCM and subprime crises. We observe the same behavior for the co-kurtosis of both indices. This suggests that risk related to higher moments is higher for the market portfolio than for hedge funds during crises. It is much more similar during expansion.

An anecdote may suggest why a smaller co-skewness and co-kurtosis for the general hedge fund index than for the market portfolio is not so surprising after all. Piketty (2013, pp. 714-719) studied the portfolio composition and the performance of U.S. universities’ endowments over the period 1980-2010. He found that the weight of alternative investments—like investments in hedge funds and structured products—increases quickly with the size of the endowment. The mean return of the endowment also increases with the size of its portfolio. Indeed, there is a barrier-to-entry for investors in the hedge fund industry. The required capital is high and so is the required manager’s expertise. That may partly explain why the risk-return trade-off of the general hedge fund index appears more favorable than the one of the market portfolio. However, return smoothing—a practice observed in the hedge fund industry—leads to an underestimation of risk as measured by standard deviation, (negative) skewness or kurtosis (Brown et al., 2012; Bali et al., 2014). When studying return smoothing, we will qualify our optimistic comments regarding hedge funds’ higher moment (systematic) risk in relation with the corresponding market risk.

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26 However, the increase in co-kurtosis was small during the burst of the tech bubble at the beginning of the second millennium. Perhaps it was better controlled by hedge funds than during the Asian-Russian-LTCM crisis which was very close to the tech bubble crisis.

27 For instance, for endowments less than 50 million (euros), the weight of alternative investments is lower than 10%. This weight jumps to 45% for endowments comprised between 500 million and 1 billion and may exceed 60% for endowments exceeding 1 billion (Piketty, 2013, p. 718).

28 For instance, for the top endowments (Harvard, Yale and Princeton whose endowments are approximately equal to $30 billion, $20 billion and $15 billion in 2010, respectively), the mean return of their portfolio was 10.2% over the period 1980-2010 and approximately the same level—i.e., 10%—for the period 1990-2010. This return is two times higher than the one obtained by universities with smaller endowments.
In this respect, a look at the behavior of the co-skewness and co-kurtosis of short-sellers and futures’ strategies from 1995 to 2012 is quite instructive (Figure 5). As just explained, the risk related to return higher moments increased substantially for the market portfolio over the recent crises. This is not the case for the short-sellers and futures’ strategies. Indeed, the co-skewness improved drastically for the futures and especially for the short-sellers’ strategy during the Asian-Russian-LTCM and the subprime crises. Risk associated with co-skewness thus decreased during crises for both strategies. Figure 5 also shows that risk associated with co-kurtosis decreased greatly during crises—and especially for the short-sellers, whose return is actually negative in good times.

It is well known that the transactions of hedge funds display an option-like behavior (Fung and Hsieh, 1997, 2001, 2004; Mitchel and Pulvino, 2001; Agarwal and Naik, 2004; Agarwal et al., 2017a,b). In this regard, Fung and Hsieh (2001, 2004) show that the returns of many hedge fund directional strategies is related to lookback straddles. In Figure 5, we compare the co-skewness and the co-kurtosis of short-sellers and futures to the respective co-skewness and co-kurtosis of the first principal component (pc_lookback) of the Fung and Hsieh (2001, 2004) five lookback straddles—i.e., lookback straddles on bonds, stocks, short-term interest rate, foreign currencies and commodities. Our pc_lookback is a global indicator of the volatility of financial, commodity and currency markets. The behavior of co-skewness and co-kurtosis of short-sellers and futures is very close to the one of pc_lookback—especially during recessions or crises. In this respect, risk associated with co-skewness and co-kurtosis decreases markedly during crises for the two strategies and the pc_lookback. Hence, the good performance of short-sellers and futures during crises is mainly explained by their hedging (short selling) activities. Short-sellers and futures seem to be good market timers. Indeed, the payoffs of a perfect market timer who is not involved in short sales are similar to the ones of a long call. Being also involved in short sales transform these payoffs in straddles or lookback straddles (Stafylas et al., 2017).

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29 According to Table 3, these two strategies display a non-significant AR(1) coefficient, even in times of crisis. This indicates that return smoothing does not greatly distort the risk associated with these two strategies. The same argument holds for the macro strategy.

30 A lookback call option gives the right to buy the underlying asset at its lowest price observed over the life of the option. A lookback put option allows the owner the sell the underlying asset at the highest price observed over the life of the option. The combination of these two options is the lookback straddle (Fung and Hsieh, 2004).
Figure 6 highlights the respective behaviors of short-sellers and futures’ strategies co-skewness and co-kurtosis during the subprime crisis. The decrease in risk related to co-skewness is relatively important. This suggests that the short-sellers and futures’ strategies may be a “good hedge” for portfolios during crises.

Insert Figure 7 here

Figure 7 plots the respective behaviors of co-skewness and co-kurtosis for four directional hedge fund strategies: long-short, growth, macro, and value index. The plots of the higher moments of the long-short and growth strategies’ returns are very similar to those of the hedge fund general index. However, the risk associated with the long-short, growth and value index strategies increases more than the general index in crises. In contrast, the risk associated with the macro strategy—as measured by the returns’ higher moments—is much less than the one of the general index during crises and is quite comparable during expansions.

Insert Figure 8 here

Figure 8 plots the co-skewness and co-kurtosis of three non-directional strategies: equity market neutral, distressed securities, and event driven. With the exception of market neutral, the plots of these strategies are similar to the ones of the general index. In contrast, the behavior of the higher moments of the equity market neutral strategy is very stable compared to the general index.

Insert Figure 9 here

Finally, it is often argued that the $SMB^{31}$ and $HML^{32}$ Fama and French (1992, 1993) factors may capture higher order systematic risk in a CAPM augmented asset pricing model (e.g., Chung et al., 2006; Hung, 2008). Figure 9 supports this conjecture. During periods of crisis, the co-skewness of $SMB$ is very close to the hedge fund general index. Although it was more volatile during expansions before 2004, the co-skewness of $SMB$ has been more correlated with the one of the general index since 2004. Note also the conformity of the co-kurtosis of $SMB$ with the general index one. More importantly, the co-skewness and co-kurtosis of $HML$ are even more correlated with the respective co-skewness and co-kurtosis of the general index. During crises, the patterns of the co-skewness and co-kurtosis of $HML$ mimic remarkably well the ones of the general index although with a greater amplitude. The

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$^{31}$ $SMB$ is the return of a portfolio which is long in small firms’ stocks (small capitalization) and short in big firms’ stocks (high capitalization).

$^{32}$ $HML$ is the return of a portfolio which is long in value stocks (high book-to-market value) and short in growth stocks (low book-to-market value).
Fama and French $SMB$ and $HML$ factors thus stand as good indicators of systematic risk embedded in the higher moments of hedge fund returns—i.e., co-skewness and co-kurtosis. As shown previously, our $pc\_lookback$ indicator also embeds a share of hedge fund co-skewness and co-kurtosis. The results reported in Tables 4 and 5 are also in line with this conjecture.

4. Empirical results

For both indicators of tail risk—i.e., co-skewness and co-kurtosis—we first discuss the GMM results and then turn to the OLS ones in order to investigate the endogeneity issues related to the presence of generated variables. To select the shocks which impact co-skewness and co-kurtosis, we rely on stepwise regressions. We retain shocks whose coefficients are significant at the 10% level and less.

Insert Table 4 and Figure 10 here

4.1 Co-skewness

According to GMM estimations, the main shocks which impact co-skewness are $cv\_gprod$, $VIX$, and $PS$. Excepting short-sellers and futures, strategies’ co-skewness react negatively to $cv\_gprod$ and $VIX$ (Table 4 and Figure 10). An increase in the uncertainty related to production growth and a rise in market volatility thus result in an increase in risk as measured by a decrease in return co-skewness.

The elasticity of the general index co-skewness to $cv\_gprod$ is equal to -0.54, which is quite high for a monthly elasticity—i.e., an increase of 1% of $cv\_gprod$ results in a decrease of 0.54% of co-skewness. The directional strategies—i.e., growth, value index, macro, and long-short—are the most sensitive to production shocks, with elasticities in the [-0.46, -0.74] range (Figure 10). The event driven strategy, and especially the distressed strategy, display much lower elasticities. The equity market neutral, short-sellers and futures strategies seem to be immunized against this kind of shock (Figure 10). Note that the t-statistics of most co-skewness coefficients are high, which suggests that the impact of a macroeconomic shock is very significant for most strategies.

Strategies’ co-skewness is even more sensitive to VIX than to $cv\_gprod$ (Figure 10). Except for short-sellers and futures, strategies’ co-skewness deteriorates when market
volatility—as measured by VIX—increases, and this impact, like the one of macroeconomic shock, is very significant. For instance, the elasticity of the general index co-skewness is equal to -1.17, which is more than the double of its elasticity to $cv_{gprod}$. Strategies whose co-skewness deteriorates the most after an increase in VIX are: equity market neutral (-1.78), growth (-1.20), and value index (-1.17) (Figure 10). Co-skewness risk of the equity market strategy is thus especially sensitive to an increase in market volatility, which suggests that this strategy may display a weaker relative performance than the other ones when VIX jumps. Strategies that display low return variance may thus be quite risky when we account for higher moment risk. In this regard, equity market neutral strategy is not so neutral after all (Duarte et al., 2007; Patton, 2009; Agarwal et al., 2017b).

In contrast to the other strategies, the co-skewness of the short-sellers and futures strategies displays a positive elasticity with respect to VIX. These elasticities are substantial, being equal to 1.64 for short-sellers and 0.79 for futures (Figure 10). Consistent with our stylized facts, these elasticities suggest that risk as measured by co-skewness decreases when the VIX increases for these strategies—i.e., during episodes of recessions or crises. At least regarding risk associated with co-skewness, these two strategies may be considered as good “hedges” during bad times. Their positive elasticity is related to their short-selling operations during periods of tensions on the financial markets. In keeping with our results, Asness et al. (2001) find that the return of the futures strategy reacts positively to the squared market return while most other strategies load negatively on this factor. The co-skewness of this strategy should thus increase following a rise in the market volatility.

We might expect that, following a rise in market illiquidity as measured by the Pástor and Stambaugh factor ($PS$, 2003), strategy co-skewness risk should increase. Surprisingly, except for short-sellers and futures, strategies’ co-skewness risk decreases after a rise in $PS$. We can thus conjecture that $PS$ stands for as an indicator of return smoothing which is associated with a reduction of tail risk in the hedge fund industry (Getmansky et al., 2004; Brown et al., 2012). Indeed, the more smoothed are returns, the higher should be return co-skewness. For the general index, the level of the elasticity of co-skewness, at 0.44, is quite high. Strategies whose co-skewness respond the most to VIX or to $cv_{gprod}$—i.e., equity market neutral, growth, macro, and value index—display relatively high elasticities with respect to $PS$ (Figure 10). Strategies’ managers thus tend to compensate risk related to macroeconomic and volatility shocks by smoothing returns. Note that the practice of return smoothing is mainly observed during episodes of recessions or crises when market illiquidity
peaks—i.e., when $PS$ displays its highest volatility (Figure 1). This observation will be documented in our VAR analyses.

The co-skewness of some strategies is sensitive to an interest rate shock—which, in the framework of our study, may be assimilated to a monetary policy shock or an unexpected change in monetary policy—but the absolute levels of the related elasticities are smaller than the ones computed for $cv_{gprod}$ or VIX (Table 5). The co-skewness of the general index deteriorates when an interest rate shock occurs, the estimated elasticity being -0.14. The co-skewness of the long-short, growth and value index—three dominant strategies—also decreases after such a shock. In contrast, the co-skewness of short-sellers improves after this kind of shock since their behavior is the opposite of most other strategies (except possibly for the futures strategy). Finally, only the co-skewness of the equity market neutral strategy reacts significantly to the credit spread. For this strategy, co-skewness decreases when the credit spread increases—i.e., with a rise in credit risk.

Insert Table 5 and Figure 11 here

4.2 Co-kurtosis

The GMM elasticities with respect to shocks associated with strategies’ co-kurtosis are provided in Table 5 and Figure 11. Here an estimated positive elasticity for a shock means that co-kurtosis deteriorates—i.e., systematic risk as measured by co-kurtosis increases—following the occurrence of the shock.

In contrast to co-skewness results, strategies’ co-kurtosis is usually more sensitive to $cv_{gprod}$ than to VIX (Figure 11). Macroeconomic uncertainty thus seems to impact more systematic fat-tail risk—i.e., risk related to rare events—than financial uncertainty. Moreover, the levels of the elasticities of co-kurtosis to shocks tend to be smaller than for co-skewness. This suggests that co-kurtosis is under better control than co-skewness. In order words, market-timing should focus more on co-kurtosis than on co-skewness during market turmoil. This conjecture appears reasonable since fat-tail risk is the main driver of downside risk in bad times and since hedge funds ought to arbitrage co-skewness and co-kurtosis (Desmoulins-Lebeault, 2006; Jurczenko and Maillet, 2006; Berg and van Rensburg, 2008; Davies et al., 2009). In this regard, episodes of macroeconomic uncertainty are less frequent than periods of peaking market volatility. Moreover, market volatility tends to persist and is therefore to some extent predictable (Busse, 1999), which is less the case for GDP growth.
These arguments may partly justify the higher sensitivity of co-kurtosis to macroeconomic
shocks in the hedge fund industry and its lower response to VIX.

Except for the futures strategy, co-kurtosis deteriorates (increases) following the
occurrence of a shock emanating from economic growth (Figure 11). The elasticity of the
general index to $cv_{gprod}$ is equal to 0.37, smaller in absolute value than the elasticity of its
co-skewness to the same variable (-0.54). The event driven and distressed strategies have the
highest positive elasticities to $cv_{gprod}$, equal to 0.93 and 0.73, respectively (Figure 11). Both
strategies also display the highest sensitivity to the VIX, the respective elasticities being 0.54
and 0.46. This suggests that these strategies may especially embed substantial fat-tail risk
during a recession. Indeed, it is during bad times that the business lines of these strategies
are the most risky (but also with high expected payoffs). Similarly to its co-skewness, the co-
kurtosis of the equity market neutral strategy does not significantly respond to $cv_{gprod}$, but
its elasticity to VIX, at 0.42, is not negligible. The co-moments of this strategy thus deteriorate
markedly in periods of rising market volatility. The macro strategy displays an opposite
profile, in the sense that its higher co-moments are not sensitive to the VIX but respond to
economic uncertainty, albeit the corresponding elasticity is not particularly important (0.28).
Other strategies for which co-kurtosis react positively to $cv_{gprod}$ and VIX—i.e., growth, long-
short, and value index—display moderate elasticity levels. They thus track better their co-
kurtosis than the distressed and event driven strategies.

Analogously to their co-skewness, the co-kurtosis of the short-sellers and futures
strategies tends to decrease following adverse shocks (Figure 11). In this regard, the risk
related to co-kurtosis decreases for the futures strategy when $cv_{gprod}$ increases while this
kind of shock does not impact significantly the co-kurtosis of short-sellers. The elasticities of
the co-kurtosis of short-sellers and futures to VIX, with respective levels of -0.44. and -0.31,
also indicate that both strategies benefit from a rise in market volatility, a result consistent
with hedge fund stylized facts.

Surprisingly, a shock emanating from inflation—as measured by $cv_{inf}$, reduces fat-
tail risk for most strategies (Table 5). For instance, the elasticity of the co-kurtosis of the
general index to $cv_{inf}$ is equal to -0.12. Strategies which react the most to this indicator are
distressed and event driven, with respective elasticities of -0.17 and -0.16. Even the futures
strategy, which usually behaves, as short-sellers, in the opposite direction relatively to the
other strategies, displays an elasticity of -0.10. A shock to inflation may thus be beneficial for
most hedge fund strategies. As argued by Piketty (2013), inflation may benefit to wealth
holders since it tends to be associated with strong economic growth. However, the inflation shock may be related to negative inflation (deflation) as it was the case during the subprime crisis. During these episodes, central banks tend to inject huge amounts of liquidity, which loosens the financial (liquidity) constraint of investors. In fact, even if the inflation shock may be related to a jump or a drop of inflation, it seems to be associated with a loosening of investors’ financial constraint in both cases. This may partly explain why fat-tail risk decreases in the hedge fund industry with the occurrence of an inflation shock.

4.3 Common factors

The SMB (size anomaly) and HML (value anomaly) variables are important factors in hedge fund return models. They thus may partly embed the co-skewness and co-kurtosis of strategies (Chung et al., 2006; Hung, 2008). To verify this conjecture, Tables 4 and 5 provide the reaction of SMB and HML co-skewness and co-kurtosis to macroeconomic and financial shocks. We note that these two factors react in the same way to shocks as strategies with positive betas—i.e., excluding short-sellers and futures. They thus partly explain the behavior of the higher moments of these strategies. Regarding co-skewness, HML responds more to cv_gprod and VIX than SMB (Figure 10). From this point of view, it is a “riskier” factor. HML also seems to be a more important source of illiquidity for hedge funds strategies since its co-skewness responds much more to PS than the one of SMB. The results are mixed for co-kurtosis. SMB reacts more to VIX while HML responds more to cv_gprod (Figure 11). Nevertheless, both respond in the same way to shocks as strategies with positive betas.

As shown previously, the pc_lookback factor is an important determinant of the co-skewness and co-kurtosis of the short-sellers and futures strategies (Figure 5). Tables 4 and 5 support this observation. The co-skewness of the pc_lookback responds to shocks as the ones of short-sellers and futures but with lower loadings. Similarly to these two strategies, it does not respond to cv_gprod and PS (Figure 10). Turning to co-kurtosis, the pc_lookback behaves once more as the two contrarian strategies—i.e., short-sellers and futures—but its co-kurtosis is more sensitive to VIX and cv_gprod (Figure 11). Interestingly, note that, in absolute value, the co-kurtosis of the pc_lookback responds more to cv_gprod than to the VIX, a result similar to the one obtained for most strategies with positive betas. This suggests that their payoffs are similar to the ones of short lookback straddles. The reaction of pc_lookback to cv_inf also supports this conjecture (Table 5). Analagously to futures, cv_inf impacts
positively pc_lookback co-kurtosis. In contrast to the futures strategies, most other ones respond negatively to this common factor.

4.4 Endogeneity biases

A comparison of OLS and GMM coefficients in Tables 4 and 5 leads us to conclude that the estimation of co-kurtosis is more impacted by endogeneity biases than co-skewness. For instance, the coefficients associated with VIX are clearly overstated for six strategies and HML. The coefficients related to cv_inf are overstated for four strategies. We observe the same situation for the PS factor. In the case of cv_gprod, the coefficients of three strategies and SMB are overstated. OLS thus tends to overstate the loadings of the factors in the co-kurtosis estimations. Turning to co-skewness, we observe only few cases of biases. For three strategies, the coefficient associated with the VIX is overstated. The coefficients associated with GMM which were analyzed in this section are thus more reliable. Hence, the greater importance of endogeneity for co-kurtosis is another evidence that hedge funds tend to track their co-kurtosis during bad times at the expense of co-skewness.

4.5 Impulse response functions

In order to examine more precisely the dynamics of strategies’ return co-skewness and co-kurtosis with respect to their explanatory variables, we compute their impulse response functions to shocks in these explanatory variables. An impulse response function is computed using a vector autoregressive system (VAR) defined as follows: \( Y_t = \Gamma + A(\epsilon) Y_{t-1} + u_t \), where \( Y_t \) is the vector of endogenous variables and \( A(\epsilon) \) is the lag operator. If the matrix of the residuals \( (u_t) \) is diagonal, the identification of the shocks associated with the variables is straightforward. For instance, if there are two variables—say a real variable and financial variable—the residuals matrix provide directly on its diagonal the respective shocks related to these variables—i.e., the real shock and the financial shock. However, the residuals are usually correlated so that the residuals matrix is no longer diagonal. In this case, we need a method to identify the shocks related to the variables of the VAR. Sims (1980) has proposed the Cholesky decomposition of the reduced form residuals’ covariance matrix. This decomposition makes the matrix triangular and the shocks are thus identified recursively. However, in this identification process, the resulting impulse response
functions (IRF)—which provide the reaction of the VAR variables to the shocks—depend on the ordering of the variables. At the top of the VAR appear the variables which react only to their contemporaneous shocks. The adjustment speed of these variables is thus low. Conversely, at the bottom of the VAR, we find the variables whose adjustment speed is relatively high. To build the IRF (Hamilton, 1994, p.318-319), we must first transform the VAR equation into its infinite moving average representation—i.e., an MA(∞) (Wold, 1938). Then, we compute the partial derivative of the MA(∞) as follows: \( \frac{\partial Y_t}{\partial \epsilon_t} = \Psi_s \), where \( \epsilon_t \) is the vector of innovations of the MA(∞) representation. The last step consists in plotting the row \( i \), column \( j \) element of \( \Psi_s \)—i.e., the \( \frac{\partial Y_{i,t+s}}{\partial \epsilon_{jt}} \). This plot is called the IRF.

In our VAR system, we have retained the three most important explanatory variables to compute the IRF of co-skewness and co-kurtosis: the conditional variance of the production growth (\( \text{cv}_\text{gprod} \)), the VIX and PS. We associate the first variable with a real shock, the second with a volatility shock, and the third with an illiquidity shock. For instance, for co-skewness, the order of the variables reads as follows:

\[ \text{co-skewness} \leftarrow \text{cv}_\text{gprod} \leftarrow \text{VIX} \]

According to this ordering, VIX is the VAR variable with the highest adjustment speed. It impacts with a lag first \( \text{cv}_\text{gprod} \) and then co-skewness. Moreover, \( \text{cv}_\text{gprod} \) impacts co-skewness with a lag. Note that the ordering of the variables has a great influence on the IRF but this impact is lower when the correlation between the VAR residuals is weak, which is the case in our experiments.

Insert Figure 12 here

Figure 12 provides the IRF of the co-skewness of the general index return to one standard deviation of three innovations associated with \( \text{cv}_\text{gprod} \), VIX, and PS. We also divide the whole sample period in expansion and recession (or crisis) episodes since we observed important asymmetries in the behavior of systematic risk according to the phase of the business cycle when analyzing stylized facts. To account for the phases of the business cycles, we define two dummies: \text{dum} \_\text{up} \ and \text{dum} \_\text{down}. The indicator variable \text{dum} \_\text{up} takes the value of 1 during months associated with an expansion and 0 otherwise, and conversely.

---

33 By the top of the VAR we mean the top of the triangularized residuals matrix.
34 See also Ghysels and Marcellino (2018).
35 We also relied on the generalized impulse response analysis which does not depend on the ordering of the variables (Pesaran and Shin, 1998). The results do not differ greatly from the ones obtained with the Cholesky decomposition.
36 For the sake of parsimony, we do not analyze the individual hedge fund strategies in this section.
for the indicator variable *dum_down*. We then multiply our three explanatory variables—i.e., *cv_gprod*, *VIX* and *PS*—by these two dummies in order to simulate the impact of the phases of the business cycle on hedge fund systematic risk associated with return higher moments. Following these modifications, equation (6) becomes:

\[
\forall i, i = 1 \text{ to } k \quad y_t = \alpha_i + (\beta_{i, cv_gprod} \times dum_{up,t}) + (\beta_{i, cv_gprod} \times dum_{down,t}) + \ldots \\
+ (\beta_{i, VIX} \times dum_{up,t}) + (\beta_{i, VIX} \times dum_{down,t}) + (\rho_{i, PS} \times dum_{up,t}) + (\rho_{i, PS} \times dum_{down,t}) + \epsilon_{y,t} + \epsilon_u
\]

(14)

As suggested by Koop et al. (1996), we rely on the generalized impulse response function to estimate equation (14) since it is *nonlinear* in *cv_gprod*, *VIX* and *PS*.

The IRF of the co-skewness related to the general hedge fund index shows that this kind of risk increases (i.e., co-skewness decreases) significantly following a one-standard deviation jump in the real shock (*cv_gprod*) (Figure 12). The reaction of co-skewness is hump-shaped as expected for stationary variables with peak effects at the fifth month. Risk slowly decreases thereafter.

More importantly, the pattern of the IRFs observed over the whole sample period is not due to expansion episodes. Actually, during expansion, a shock has no significant impact on the co-skewness of the general index return. It is during recessions that the hump-shaped pattern observed during the whole sample period takes place. During recessions, the increase in risk—as measured by co-skewness—due to a real shock is important and very significant during the first months of the impact. Its decrease is very persistent and slow thereafter. This behavior is in line with the standard reaction of the most important macroeconomic variables—as unemployment—to a negative real shock. At the impact, the increase in unemployment is high but its decrease is very slow thereafter, as observed during the subprime crisis. Overall, there is an obvious asymmetry in the IRF behavior according to the phase of the business cycle. As Dewachter and Wouters (2014) explain, this asymmetry relates to the fact that “the reaction of agents is much more pronounced during periods of recession while they behave much smoothly during booming business cycle periods”.

The IRFs associated with a one standard deviation increase in the financial shock (*VIX*) are similar to the ones observed for the real shock. There is no obvious reaction of the

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37 Indeed, the confidence interval of the IRF is under the 0 level until the tenth month of the simulation.

38 *Deleveraging* is one of the main techniques used by financial institutions to reduce risk following a real or financial negative shock. But the impact of this process on risk is relatively slow, which may explain the long-lasting decline in systematic risk in our IRF plots (Calmès and Théoret, 2013). Financial institutions also rely on asset sales to strengthen up their financial results in recessions but they rarely rely on the issue of financial capital in recession to strengthen their balance sheet because agency problems and asymmetries of information are much higher in recessions than in expansions. The financial accelerator increases the cost of external finance in recession, which hampers the issue of new stocks by firms (Bernanke et al., 1999).
risk associated with co-skewness during expansion (Figure 12). However, similarly to the real shock, the IRF plot during recession shows that risk increases very quickly following a financial shock and peaks earlier than the real shock, a sensitive result since the adjustment speed is reasonably quicker for a financial shock than for a real shock. Turning to PS, we note that its IRF is significant at the 10% level in recession. It peaks very quickly and it is not persistent thereafter, suggesting, as expected, that return smoothing is not a lasting practice.

The variance decomposition shows that the percentage of the variance of co-skewness explained by the VIX and cv_gprod are both important. However, the percentage explained by the VIX (36%) is greater than the one due cv_gprod (29%)(Figure 13). This result is consistent with our GMM estimations which give a greater weight to the real shock over the financial shock in the explanation of risk related to the general index co-skewness. The PS shock has a low impact in the variance decomposition over the whole sample period (1%) but its influence is higher over the recession period (5%).

We obtain results similar to the general index co-skewness when computing the IRFs of its co-kurtosis—i.e., systematic fat-tail risk (Figure 14). Namely, shocks do not impact co-kurtosis during an expansion. During a recession, a real shock leads to a significant increase in the risk related to co-kurtosis—i.e., a jump in co-kurtosis during the first months of the simulation. Similarly to co-skewness, this jump is followed by a smooth decrease in fat-tail risk. The pattern of the IRFs’ co-kurtosis is thus consistent with the general pattern of the macroeconomic aggregates’ IRFs. Furthermore, we observe the same pattern for the behavior of the IRFs of the general index co-kurtosis following VIX, although its impact is quicker. The impact of a PS shock is also mainly at play in recession, and, consistent with co-skewness results, its impact is quick and not persistent. The variance decomposition (Figure 15) is comparable to the one obtained for co-skewness.

4.6 Robustness checks
4.6.1 Considering another illiquidity factor: the Amihud ratio

In order to better grasp the role of illiquidity on hedge fund strategies’ higher moments, we consider another illiquidity indicator in addition to PS: the Amihud ratio. The

Note that the variance decomposition should converge towards the percentages adjusted for return smoothing since the lagged dependent variables are introduced in the simulation.
Amihud (2002) illiquidity ratio is the daily ratio of absolute stock return to its trading volume, averaged over each month, i.e.,

\[ LIQ_{Amihud} = \frac{1}{D_i} \sum_{i=D_i}^{D} \frac{|R_i|}{Vol_i} \]  

(15)

where \( D_i \) is the number of days of the month, \( R_i \) is the daily return on stock \( i \) and \( Vol_i \) is its corresponding trading volume. In this paper, the Amihud (2002) illiquidity measure is computed using the S&P500. The Amihud ratio quantifies the price/return response to a given size of trade. According to Naes et al. (2010), this ratio is a measure of the elasticity dimension of liquidity, in the sense that it tries to capture the sensitivity of prices to trading volume. When the Amihud ratio is high, liquidity is low. As argued by Konstantopoulos (2016), in most research papers, the coefficients of the \( PS \) and Amihud illiquidity factors are positive in return estimations. Indeed, these factors are proxies for the illiquidity premium, which is a component of returns. However, according to Acharya and Pedersen (2005), a positive shock in illiquidity predicts high future illiquidity, so contemporaneous stock prices decrease, which can result in a negative sign for the coefficients of the \( PS \) and Amihud factors.

In our sample, the Amihud ratio has no significant correlation with \( PS \) but a correlation of 0.27 with the VIX. Table 6 shows the elasticities of strategies co-skewness and co-kurtosis to the Amihud ratio estimated with OLS and GMM\(^40\). Even if there is no significant correlation between \( PS \) and the Amihud ratio, the elasticities obtained with GMM are consistent with the ones associated with \( PS \). They tend to be positive for co-skewness and negative for co-kurtosis, which suggests that the illiquidity factor is related to return smoothing. Once more, short-sellers and futures display an opposite reaction to the Amihud ratio.

Interestingly, these estimations reveal an important endogeneity issue. Indeed, elasticities are not significant for any strategy when using OLS, but are significant for most of them at the 10% level or less when relying on GMM, which supports the conjecture that liquidity is endogenous (Brunnermeier and Sannikov, 2014; Adrian et al., 2017).

4.6.2 Tackling the problem of return smoothing

4.6.2.1 Computing the smoothing coefficients

Theoretically, there should be no autocorrelation between returns if financial markets were perfect because, otherwise, returns would be predictable. However, market

\(^{40}\)To compute the elasticities of Table 6, we relied on our complete higher moment model. However, for the sake of simplicity, we only report the estimated elasticities associated with the Amihud ratio.
frictions like return smoothing and illiquidity can lead to autocorrelation in the return series (e.g., Getmansky et al., 2004; Chan et al., 2007; Brown et al., 2012; Bali et al., 2014). But whatever the sources of autocorrelation, this problem reduces observed systematic risk and it appears to be particularly important in the hedge fund industry. In this respect, Brown et al. (2012) find that return smoothing may reduce substantially funds of hedge funds’ total risk as measured by variance, skewness and kurtosis.

In our previous empirical analysis, we accounted for this phenomenon by adding the lagged dependent variables in our estimations. Return smoothing was associated with a delay in the adjustment of hedge fund risk measures to their effective (unobserved) values. We think that this is the best way to proceed initially because, as we show, there are many ways to adjust return series for smoothing and the results may be quite different. In this section, we investigate how return smoothing may modify our previous results. To anticipate, return smoothing may greatly reduce systematic risk associated with return higher moments, and especially fat tail risk, an obvious bias in the estimation process. However, this practice is not persistent and is mainly at play during crises.

In this section, we account for return smoothing by relying on three procedures:

(i) Since the formulas of unsmoothed covariances are only valid asymptotically and since measures of systematic risk are relative, we first smooth the higher moments of the market return with the vector of (smoothing) coefficients estimated for the hedge fund general index;

(ii) Second, we compute the *correction factor* (multiplier) of the smoothed covariance measures by calculating the asymptotic covariances related to co-skewness and co-kurtosis. Using the market model for computing the vector of the smoothing coefficients, we recover the unsmoothed series of co-skewness and co-kurtosis of the hedge fund general index;

(iii) We repeat this procedure using another model—i.e., the moving average (MA(k)) model, as suggested by Getmansky et al. (2004) and Brown et al. (2012).

To implement these three procedures, we first compute the coefficient vector used to smooth the series. We rely on the two methods proposed by Getmansky et al. (2004) to compute this vector. We first resort to the following market model to compute it:

\[
R_t = \beta_0 + \beta_1 R_{m_t} + \beta_2 R_{m_{t-1}} + \ldots + \beta_k R_{m_{t-k}} + \epsilon_{it} \quad (15)
\]

These asymptotic assumptions may result in a decrease in effective risk, another source of bias in reduced samples.
where \( R_{it} \) is the hedge fund return, \( R_{mt} \) is the return on the market portfolio (market return), and \( \varepsilon_{it} \) is the innovation.

The smoothed hedge fund return series is the following moving average of the unsmoothed return series:

\[
R_{it} = \theta_1 R^*_t + \theta_2 R^*_t + \ldots + \theta_k R^*_t \tag{16}
\]

where \( R^*_t \) is the unsmoothed (unobserved) return. The parameters of equation (16) are computed using equation (15) (Getmansky et al., 2004):

\[
\theta_i = \frac{\beta_i}{\sum_{i=1}^{k} \beta_i} \tag{17}
\]

The constraint on the \( \theta \) coefficients—i.e., that their sum must be equal to 1—is thus obviously satisfied.

The second method proposed by Getmanky et al. (2004) to compute the vector \( [\theta] \) is to estimate an MA(\( k \)) process by imposing the constraint that the sum of the coefficients must be equal to 1. This equation is estimated using the constrained maximum likelihood. It provides directly the vector of the \( \theta_i \) coefficients.

According to the market model method, the vector of \( \theta \) parameters is equal to: \([0.76 \ 0.15 \ 0.09]\), corresponding to \([\theta_1 \ \theta_2 \ \theta_3]\) in equation (16), while the MA(3) method provides the corresponding MA vector: \([0.44 \ 0.30 \ 0.26]\). The lags are selected according to the classical information criteria (Akaike, Schwarz). The estimation of the parameters thus differs according to the method used. The MA method provides greater smoothing than the market model.

Insert Figure 16 here

It is interesting to run a recursive regression on the MA(3) process in order to examine the cycles of the smoothing coefficient \( \theta_1 \) which will be used to retrieve the unsmoothed co-skewness and co-kurtosis. Figure 16 shows that this coefficient tends to decrease at the beginning of a recession—the managers of hedge fund portfolios smooth returns at the start of a recession. However, it rebounds quickly—i.e., before the recovery of the general index. As expected, return smoothing is not a practice which tends to persist and it is mainly at play during crises. Return smoothing is thus countercyclical.

4.6.2.2 Adjusting the return variance for smoothing.
To adjust variance, we use directly the multiplier proposed in Getmansky et al. (2004) which is equal to: \( \frac{1}{\theta_1^2 + \theta_2^2 + \theta_3^2} \). According to the coefficients computed with the market model, this increases the smoothed variance by a factor of 1.67. Conversely, according to the MA(3) variance, the multiplier is 2.84.

Insert Figure 17 here

Figure 17 plots the corresponding adjustments to the general index variance. We note that the general index variance is greatly understated during the tech-bubble crisis. It is even greater than the market return variance during this period. But the downward trend of the general index variance is observed thereafter even after adjusting for smoothing. Even if, as expected, the variance of the general index is greater using the MA method than the market model one, it remains substantially less than the market return one during the subprime crisis. The tendency of the hedge fund return variance to be lower than the market one is also observed by Brown et al. (2012) for the case of funds of hedge funds.

Insert Figure 18 here

Another way to compare the variances of the market return and the general index return is to let the variance of the general index unadjusted but to compute the variance of the market return smoothed by the two sets of coefficients computed in the previous section. For simplicity, for the remainder of this section, we will only discuss the adjustment performed by the MA model since the market model always provides less risk. Once more, the variance of the market return is usually greater than the general index one even if smoothing greatly reduces the market return variance compared to the general index (Figure 18). However, during the tech-bubble crisis, the variance of the general index was greater than the smoothed variance of the market return, while it was the opposite on an unadjusted basis. The risk of hedge funds thus seems to have been understated during this period.

Insert Figures 19 and 20 here

4.6.2.3. Smoothing co-skewness and co-kurtosis of the market return

As explained before, to capture the impact of smoothing on the general index co-skewness and co-kurtosis, we first smooth the market return series with the coefficient vectors computed for the general index. Figure 19 plots the adjustment for co-skewness and Figure 20, for co-kurtosis.

With smoothing (MA coefficients), market return co-skewness is much closer to the general index but it remains lower during the subprime crisis. However, co-skewness risk is
reduced to the level of the general index during the tech-bubble crisis. As mentioned previously, it seems that hedge funds’ managers were involved in a learning process from one crisis to the next in the sense that they learned to better control risk as measured by co-skewness. However, the effect of smoothing on co-kurtosis is drastic (Figure 20). Without smoothing, the co-kurtosis of the market return is always higher than the general index one, especially during crisis periods. This is quite suspicious because it is usually thought that fat-tail risk is higher for hedge funds than for the market portfolio, at least when fat-tail risk is gauged by kurtosis. When the market return is smoothed, the co-kurtosis of the market return is less than the general index one. According to Figure 20, fat-tail risk was much lower for hedge funds than for the market portfolio before correction for smoothing during the tech-bubble crisis and much higher after correction, which suggests that smoothing exerts a great impact particularly on the level of fat-tail risk.

4.6.2.4 Unsmoothing the general index co-skewness and co-kurtosis with statistical procedures

In this section, we retrieve the unsmoothed series of the general index co-skewness and co-kurtosis. The hedge fund return smoothed series is a moving average of the unsmoothed (unobserved) corresponding returns:

\[ R^*_t = \theta_1 R^*_t + \theta_2 R^*_t \eta_{t-1} + \ldots + \theta_k R^*_t \eta_{t-k} \]  (18)

The observed co-skewness is given by \( \text{Cov}(R^*_t, R^2_{mt}) \). Substituting (18) in this expression, we have:

\[ \text{co-skewness} = \text{Cov}(\theta_1 R^*_t, \theta_2 R^*_t \eta_{t-1} + \ldots + \theta_k R^*_t \eta_{t-k}, R^2_{mt}) \]  (19)

\[ = \text{Cov}(\theta_1 R^*_t, R^2_{mt}) + \text{Cov}(\theta_2 R^*_t \eta_{t-1}, R^2_{mt}) + \ldots + \text{Cov}(\theta_k R^*_t \eta_{t-k}, R^2_{mt}) \]  (20)

where we use lowercase letters to express the return variables in deviation from their mean.

Since the \( r^*_t \) series is not autocorrelated, all the \( E(r^*_{t-j} r^2_{mt}) \) are equal to 0 for \( j>0 \). Equation (20) thus reduces to:

\[ = E(\theta_1 r^*_t r^2_{mt} + \theta_2 r^*_t \eta_{t-1} r^2_{mt} + \ldots + \theta_k r^*_t \eta_{t-k} r^2_{mt}) = \theta_1 E(r^*_t r^2_{mt}) \]  (21)

\[ \text{Getmansky et al. (2004) provides equivalences between smoothed and unsmoothed risk measures only for return variance, beta and Sharpe ratios.} \]

\[ \text{To obtain this expression, we rely on the basic rule on covariances: Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z).} \]

\[ \text{Note that these expectations are equal to 0 only asymptotically. This explains why we previously smoothed the market return series to avoid this constraint since our sample is not infinite. We observed that it may matters much in terms of risk.} \]
In fact, co-skewness is equal asymptotically to the Black’s leverage effect (1976), which assesses that bad news increases more market volatility than good news. Hence, the relationship between the smoothed co-skewness and the unsmoothed one is:
\[
\text{cov}(r^*_i, r^2_{mt}) = \theta_1 \text{cov}(r^*_i, r^2_{mt}) \quad (22)
\]
To compute the unsmoothed co-skewness, we thus multiply the smoothed co-skewness by \(\frac{1}{\theta_1}\), a factor greater than one since \(\theta_1 < 1\). Using the same procedure, to compute the unsmoothed co-kurtosis, we adjust the smoothed co-kurtosis with the same multiplier \(\frac{1}{\theta_1}\).

Insert Figures 21 and 22 here

Using this approach, Figure 21 compares the smoothed and unsmoothed co-skewness of the hedge fund general index to the corresponding co-skewness of the market return. We note that hedge fund systematic skewness risk may be greatly understated relative to the market portfolio, especially when adjusting it with the MA model for which smoothing has the greatest impact. However, during the subprime crisis, this kind of risk, even if it increases substantially after correction, remains lower for hedge funds than for the market portfolio. Note also that co-skewness risk is greatly reduced for the market portfolio during periods of economic expansion. This is not the case for the hedge fund co-skewness, which shows a tendency to remain negative during good times. This result is in line with many previous studies (e.g., Sabbaghi, 2012). Similar results are obtained for co-kurtosis when using our statistical multipliers to recover unsmoothed hedge fund co-kurtosis. However, this kind of risk tends to remain higher for the market portfolio regardless of the multiplier used. Note that adjustments to risk performed in Figures 21 and 22 do not fundamentally alter our previous empirical results since the hedge fund higher moments series are adjusted proportionally. However, they increase the amplitude of the elasticities of higher moment risk with respect to economic macroeconomic and financial shocks45.

5. Conclusion

Studies relating risk relating the higher moments of hedge fund returns to macroeconomic and financial shocks are sparse. Our paper attempts to fill this void in the

45 Note that in the partial adjustment model used in the empirical section (equation (6)), we compute the short-run elasticities but it is possible to recover long-run elasticities which are adjusted for return smoothing. See Greene (2000), p.722.
current literature by analyzing hedge fund tail risk in a macro-econometric framework. More precisely, we aim at studying the response of hedge fund strategies’ co-skewness and co-kurtosis to macroeconomic and illiquidity factors while accounting for the endogeneity issue associated with the computation of shocks and the behavior of hedge funds. We then transpose our model to a dynamic setting by relying on a VAR process which incorporates the phases of the business cycle. This allows us to investigate the persistence of the various shocks on hedge fund tail risk.

Our investigations lead to major empirical results which are not documented in previous studies. First, uncertainty impacts co-skewness and co-kurtosis mainly in recessions. Its effect is generally not significant in expansions. Second, hedge funds monitor their co-kurtosis at the expense of co-skewness—especially during recessions or crises. This behavior appears reasonable since the control of co-kurtosis—the main source of downside risk—is more important during crises than the monitoring of co-skewness. Co-kurtosis is also more subject to return smoothing than co-skewness. Third, the illiquidity factors—i.e., PS and the Amihud ratio—are indicators of return smoothing in the sense that an increase in illiquidity tends to decrease hedge fund tail risk. Fourth, endogeneity is an important issue in our empirical results. In this regard, co-kurtosis is more subject to endogeneity than co-skewness, endogeneity being related to the market-timing behavior of hedge funds. This is another evidence that hedge funds focus more on the monitoring of their co-kurtosis. Another result related to endogeneity is that the estimated elasticities for the Amihud ratio are not significant when using OLS but become significant when using GMM. Fifth, the co-kurtosis of most strategies responds more to production shocks than to VIX ones, suggesting that financial shocks are more manageable since they can be forecasted more easily. In contrast, co-skewness is more responsive to VIX shocks since hedge funds tend to track less this measure of risk—especially during crises. Sixth, our VAR analyses reveal interesting results regarding the persistence of shocks. Output shocks are more persistent than VIX ones, which suggests that financial shocks are more predictable. Moreover, illiquidity shocks reverse quickly, which signals that return smoothing is not a persistent practice.

References


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# Tables

**Table 1** Description of the variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition and construction</th>
</tr>
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<tbody>
<tr>
<td>VIX</td>
<td>Implied volatility of the S&amp;P500 stock index</td>
</tr>
<tr>
<td>credit_spread</td>
<td>Spread between BBB and AAA returns on U.S. corporate bonds</td>
</tr>
<tr>
<td>PS</td>
<td>The non-traded liquidity factor is an average of the stock ( \hat{\gamma}<em>{it} ) obtained from the following regression: ( r</em>{itd} - r_{idt} = \theta_i + \varphi r_{it} + \gamma_i \text{sign}(r_{it} - r_{id}) V_{idt} + \epsilon_{itd} ), where ( r_{itd} ) is the return of stock ( i ) on day ( d ) in month ( t ) and ( V_{idt} ) is the dollar trading volume of stock ( i ) on day ( d ) in month ( t ). Since illiquidity is greater when ( \hat{\gamma}_{it} ) becomes more negative, this factor is a measure of liquidity. There also exists the tradable illiquidity factor which is a mimicking portfolio being long in illiquid assets and short in liquid ones. This last measure constitutes our PS indicator.</td>
</tr>
<tr>
<td>cv_gprod</td>
<td>The conditional variance of industrial production growth. It is the conditional variance of the innovation of an ARMA(2,2) process—i.e., the mean equation—applied to the industrial production growth time series. This conditional variance is computed using an EGARCH process. It represents the second moment of the industrial production growth (first moment).</td>
</tr>
<tr>
<td>cv_gprod_w</td>
<td>The weighted conditional variance of the growth of industrial production. It is a weighted average of cv_gprod computed on a rolling window of four months with declining weights of 0.4, 0.3, 0.2 and 0.1</td>
</tr>
<tr>
<td>cv_inf</td>
<td>The conditional variance of inflation. It is the conditional variance of the innovation of an AR(1) process—i.e., the mean equation—applied to the inflation time series. The conditional variance is computed using an EGARCH process. It represents the second moment of inflation (first moment).</td>
</tr>
<tr>
<td>cv_tb</td>
<td>The conditional variance of the first difference of the U.S. three-month Treasury Bill rate. It is the conditional variance of the innovation of an ARIMA(1,1,1)—i.e., the mean equation—applied to the T-Bill rate series. The conditional variance is computed using an EGARCH process.</td>
</tr>
<tr>
<td>cv_tb_w</td>
<td>The weighted conditional variance of the first difference of the U.S. three-month Treasury Bill rate. It is a weighted average of cv_tb computed on a rolling window of four months with declining weights of 0.4, 0.3, 0.2 and 0.1</td>
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<td>cv_rsp500</td>
<td>The conditional variance of the S&amp;P500 return. It is the conditional variance of the innovation of an ARMA(1,1)—i.e., the mean equation—applied to the S&amp;P500 return series. The conditional variance is computed using an EGARCH process.</td>
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<tr>
<td>cv_gmp</td>
<td>The conditional variance of the first difference of the logarithm of a weighted composite index of commodity prices. It is the conditional variance of the innovation of an AR(1) process—i.e., the mean equation—applied to weighted commodity prices index. The conditional variance is computed using an EGARCH process.</td>
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<tr>
<td>cv_rexch</td>
<td>The conditional variance of the first difference of the logarithm of a weighted composite index of the U.S. exchange rate. It is the conditional variance of an AR(3)—i.e., the mean equation—applied to the U.S. exchange rate series. The conditional variance is computed using an EGARCH process.</td>
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**Table 2** Macroeconomic and financial shock correlation matrix
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Notes: The description of the variables is given in Table 1.
Hedge fund return descriptive statistics

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Notes: gi is the return on the GAI general index. The list of the selected strategies is: ls (long-short); macro; gr (growth); vi (value index); ds (distressed securities); ed (event driven); emn (equity market neutral); ss (short-sellers); fut (futures). The mean return of the strategies is computed over the whole sample period and also over the following subperiods in order to highlight the episode of the subprime crisis: January 1995 – May 2007 (before the subprime crisis); June 2007 – December 2009 (during the subprime crisis), and January 2010 – September 2012 (after the subprime crisis). sd stands for standard deviation and skew for skewness. The Sharpe index is equal the mean excess return of a strategy divided by the return standard deviation. The AR(1) coefficients are obtained by regressing a series on its first-order lag. The starred coefficients are significant at the 5% level.
Table 4 OLS and GMM estimations of the response of hedge fund return co-skewness elasticities to shocks

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**Notes:** See Tables 1 and 3 for the definition of the variables. Coefficients’ t-statistics are in italics. Elasticities are obtained with the following formula: \( \beta \times \frac{X}{Y} \). SMB is the size factor; HML is the value factor. pc_lookback is the first principal component of the five Fung and Hsieh (2001) lookback straddles.
**Table 5** OLS and GMM estimations of the response of hedge fund return co-kurtosis elasticities to shocks

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<th>PS</th>
<th>adj. R²</th>
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Notes: See Tables 1 and 3 for the definition of the variables. t-statistics are in italics. Elasticities are obtained with the following formula: \( \beta \times \frac{X}{Y} \). SMB is the size factor; HML is the value factor. pc_lookback is the first principal component of the five Fung and Hsieh (2001) lookback straddles.

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**Table 6** OLS and GMM estimations of the elasticities of strategies’ co-skewness and co-kurtosis to the Amihud ratio
Notes: Coefficients which are significant at the level of 10% and less are shaded. This table is built using the GMM specifications appearing in Tables 4 and 5, respectively for co-skewness and co-kurtosis, where PS is replaced by the Amihud (2002) ratio.
**Figures**

**Figure 1** Indicators of macroeconomic and financial shocks vs the output gap

*cv_gprod, cv_inf, cv_tb, cv_rsp500, VIX, credit spread, PS*

*Notes:* The output gap is computed as follows. We first take the log of the industrial production. We then detrend this transformed series with the Hodrick-Prescott filter using a smoothing coefficient ($\lambda$) equal to 14400—the trend of the series being a measure of potential output. The resulting residuals are the output gap measure. Shaded areas are associated with periods of economic slowdown. See Table 1 for the description of the variables.
**Figure 2** Rolling standard deviations and conditional covariances with the VIX: GAI general index return and S&P 500 return

**Panel A**
Rolling standard deviations of indices

**Panel B**
Conditional covariances of indices with the VIX

*Notes.* The standard deviations are computed on a rolling window of twelve months. The conditional covariances are computed with a MGARCH based on the BEKK procedure (Bollerslev et al., 1988; Engle and Kroner, 1995).

**Figure 3** Co-skewness and co-kurtosis of the hedge fund general index vs the output gap

**co-skewness**

**co-kurtosis**

*Notes:* The output gap is computed as follows. We first take the log of the industrial production. We then detrend this transformed series with the Hodrick-Prescott filter using a smoothing coefficient (λ) equal to 14400—the trend of the series being a measure of potential output. The resulting residuals are the output gap measure. Shaded areas are associated with periods of economic slowdown. MOVCOV_GI2 is the gi’s co-skewness. MOVCOV_GI3 is the gi’s co-kurtosis.

**Figure 4** Co-skewness and co-kurtosis of the hedge fund general index vs the S&P500

**co-skewness**

**co-kurtosis**
Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_MKT2 is market’s co-skewness and MOVCOV_MKT3 is market’s co-kurtosis.

**Figure 5** Co-skewness and co-kurtosis of the short-sellers and futures strategies vs the general index and lookback straddle

**Short-sellers**

<table>
<thead>
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<th>co-kurtosis</th>
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</thead>
</table>

Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_SS2 is short-sellers’ co-skewness and MOVCOV_SS3 is short-sellers’ co-kurtosis. MOVCOV_PLOOKBACK2 is the coskewness on the first principal component of the five Fung and Hsieh (2001) lookback straddle time series.

**Futures**

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<th>co-kurtosis</th>
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</thead>
</table>

50
Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_FUT2 is futures’ co-skewness and MOVCOV_FUT3 is futures’ co-kurtosis. MOVCOV_PCLOOKBACK3 is the co-kurtosis on the first principal component of the five Fung and Hsieh (2001) lookback straddle time series.

Figure 6 Co-skewness and co-kurtosis of the short-sellers and futures strategies during the subprime crisis

Short-sellers

Futures

Notes: MOVCOV_SS2: co-skewness MOVCOV_SS3: co-kurtosis

Notes: MOVCOV_FUT2: co-skewness MOVCOV_FUT3: co-kurtosis

Figure 7 Co-skewness and co-kurtosis of directional strategies vs the general index

Long–short
Notes: MOVCOV_GI2 is GI's co-skewness and MOVCOV_GI3 is GI's co-kurtosis. MOVCOV_LS2 is long-short's co-skewness and MOVCOV_LS3 is long-short's co-kurtosis.

Growth

Notes: MOVCOV_GI2 is GI's co-skewness and MOVCOV_GI3 is GI's co-kurtosis. MOVCOV_GR2 is growth's co-skewness and MOVCOV_GR3 is growth's co-kurtosis.

Macro

Notes: MOVCOV_GI2 is GI's co-skewness and MOVCOV_GI3 is GI's co-kurtosis. MOVCOV_GR2 is growth's co-skewness and MOVCOV_GR3 is growth's co-kurtosis.
Notes: MOVCOV_GI2 is gi's co-skewness and MOVCOV_GI3 is gi's co-kurtosis. MOVCOV_MACRO2 is macro's co-skewness and MOVCOV_MACRO3 is macro's co-kurtosis.

**Value index**

**co-skewness**

Notes: MOVCOV_GI2 is gi's co-skewness and MOVCOV_GI3 is gi's co-kurtosis. MOVCOV_VI2 is value index's co-skewness and MOVCOV_VI3 is value index's co-kurtosis.

**co-kurtosis**

Notes: MOVCOV_GI2 is gi's co-skewness and MOVCOV_GI3 is gi's co-kurtosis. MOVCOV_VI2 is value index's co-skewness and MOVCOV_VI3 is value index's co-kurtosis.

**Figure 8** Co-skewness and co-kurtosis of non-directional strategies vs the general index

**Equity market neutral**

**co-skewness**

**co-kurtosis**
Notes: MOVCOV_GI2 is gi's co-skewness and MOVCOV_GI3 is gi's co-kurtosis. MOVCOV_EMN2 is equity market neutral’s co_skewness and MOVCOV_EMN3 is equity market neutral’s co-kurtosis.

**Distressed Securities**

co-skewness  
co-kurtosis

Notes: MOVCOV_GI2 is gi's co-skewness and MOVCOV_GI3 is gi's co-kurtosis. MOVCOV_DS2 is distressed securities’ co-skewness and MOVCOV_DS3 is distressed securities’ co-kurtosis.

**Event driven**

co-skewness  
co-kurtosis
Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_IED2 is event driven’s co-skewness and MOVCOV_IED3 is event driven’s co-kurtosis.

Figure 9 Co-skewness and co-kurtosis of SMB and HML vs the general index
Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_SMB2 is SMB’s co-skewness and MOVCOV_SMB3 is SMB’s co-kurtosis.

HML

Notes: MOVCOV_GI2 is gi’s co-skewness and MOVCOV_GI3 is gi’s co-kurtosis. MOVCOV_HML2 is HML’s co-skewness and MOVCOV_HML3 is HML’s co-kurtosis.
Figure 10 Elasticities of strategies and factors co-skewness to $cv_{prod}$, VIX, and $PS$ shocks.

Note: This figure is built using Table 4.
Figure 11 Elasticities of strategies and factors co-kurtosis to $cv_{prod}$, VIX, and PS shocks.

Note: This figure is built using Table 5.
**Figure 12** IRFs of the hedge fund general index co-skewness

Response to one standard deviation of co-skewness to $cv_{gprod}$

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<th>Recession</th>
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Response to one standard deviation of co-skewness to VIX

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<tbody>
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Response to one standard deviation of co-skewness to PS

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**Notes:** The whole sample period is divided into expansion and recession periods with the help of dummy variables accounting for the phases of the business cycle. The Asian-Russian-LTCM crisis which occurred in 1997-1998 is also included in the recession episodes. The IRFs’ confidence intervals (dotted lines) were built using the analytic method (Hamilton, 1994, section 11.7). We also experimented Monte Carlo method (Hamilton, section, 11.7) to build these intervals but there was no significant differences between the results.
Figure 13 Variance decomposition of hedge fund general index co-skewness

<table>
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<th>% explained by co-skewness</th>
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<th>% explained by VIX</th>
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Notes: The confidence intervals (dotted lines) were built using the analytic method (Hamilton, 1994, section 11.7). We also experimented Monte Carlo method (Hamilton, 1994, section 11.7) to build these intervals but there was no significant differences between the results.
**Figure 14** IRFs of the hedge fund general index co-kurtosis

Response to one standard deviation of co-skewness to *cv.gprod*

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Response to one standard deviation of co-skewness to *VIX*

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Response to one standard deviation of co-skewness to *PS*

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*Notes:* The whole sample period is divided into expansion and recession periods with the help of dummy variables accounting for the phases of the business cycle. The Asian-Russian-LTCM crisis which occurred in 1997-1998 is also included in the recession episodes. The IRFs’ confidence intervals (dotted lines) were built using the analytic method (Hamilton, 1994, section 11.7). We also experimented Monte Carlo method (Hamilton, 1994, section 11.7) to build these intervals but there was no significant differences between the results.
Figure 15 Variance decomposition of the hedge fund general index co-kurtosis

% explained by co-kurtosis | % explained by cv_gprod | % explained by VIX

Notes: The confidence intervals were built using the analytic method (Hamilton, 1994, section 11.7). We also experimented Monte Carlo method (Hamilton, 1994, section 11.7) to build these intervals but there was no significant differences between the results.

Figure 16 Time-varying smoothing coefficient ($\theta_t$) of the hedge fund general index

Notes: This plot results from a recursive regression performed on a MA(3) process with the de-meaned general index return as dependent variable.

Figure 17 Adjusted (unsmoothed) hedge fund general index return variance: market model and MA(3)
Note: The unsmoothed variance is computed by applying the multiplier \( \frac{1}{\theta_1^2 + \theta_2^2 + \theta_3^2} \) to the smoothed variance. The coefficients entering in the multiplier are computed with the market model and the MA(3) model, respectively, as explained in section 4.6.
**Figure 18** Smoothed market return variance: market model and MA(3)

![Market Model vs. MA(3)](image)

*Note:* The smoothed market variance is based on the smoothed market return series. The autoregressive coefficients used are computed with the market model and the MA(3) model, as explained in section 4.6.

**Figure 19** Smoothed market return co-skewness: market model and MA(3)

![Market Model vs. MA(3)](image)

*Notes:* The smoothed market return co-skewness is based on the smoothed market return series. The autoregressive coefficients used are computed with the market model and the MA(3) model, as explained in section 4.6.
Figure 20 Smoothed market return co-kurtosis: market model and MA(3)

![Graph showing smoothed market return co-kurtosis for Market model and MA(3).]

Notes: The smoothed market return co-kurtosis is based on the smoothed market return series. The autoregressive coefficients used are computed with the market model and the MA(3) model, as explained in section 4.6.

Figure 21 Unsmoothed hedge fund general index co-skewness: market model and MA(3)

![Graph showing unsmoothed hedge fund general index co-skewness for Market model and MA(3).]

Note: The unsmoothed hedge fund general index co-skewness is computed by applying the multiplier $\frac{1}{\theta_i}$ to the smoothed corresponding series. The coefficient entering in this multiplier is computed using two models: the market model and the MA(3) model, as explained in section 4.6.
Figure 22 Unsmoothed hedge fund general index co-kurtosis: market model and MA(3)

Note: The unsmoothed hedge fund general index co-kurtosis is computed by applying the multiplier $\frac{1}{\theta_i}$ to the smoothed corresponding series. The coefficient entering in this multiplier is computed using two models: the market model and the MA(3) model, as explained in section 4.4.